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RESTRICTED L1 ESTIMATORS AND THEIR COVARIANCES,(U)
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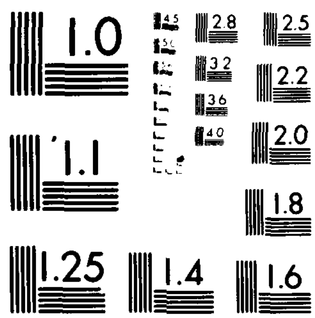

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TEXAS A&M UNIVERSITY
PROJECT THEMIS

Technical Report #65

RESTRICTED L_1 ESTIMATORS AND THEIR COVARIANCES

by

Book, D., Booker, J.,
Hartley, H.O., and Sielken, R.L. Jr.

Texas A&M University
Office of Naval Research
Contract N00014-78-C-0426
Project NR047-179

ATTACHMENT I

RESTRICTED L_1 ESTIMATORS

AND THEIR COVARIANCES

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THEMIS OPTIMIZATION RESEARCH PROGRAM
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INSTITUTE OF STATISTICS
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ATTACHMENT II

ABSTRACT

The parameters in a linear regression model can be estimated by minimizing the sum of the absolute residuals (L_1 estimation) instead of the more classical approach of minimizing the sum of squared residuals (least squares estimation). In addition to other nice properties L_1 estimators are less sensitive to outliers than least squares estimators. This paper describes a linear programming algorithm and computer program for obtaining L_1 estimators and estimates of their covariances when the regression parameters are restricted to satisfy specified linear constraints. These estimated covariances are the new feature in this work and are an extremely important ingredient in hypothesis tests and confidence interval construction. Technical Report 64 describes a similar procedure for obtaining <u>unbiased</u> L_1 estimators when there are no constraints on the parameters.

Table of Contents

Restricted L_1 Estimators and their Covariances

1. An Introduction to MR. A

Consider the linear regression model in the form

$$y = X\beta + \varepsilon \tag{1}$$

where y is a vector of n observations, X is an n x p matrix of rank p of known constants, β is a vector of p unknown parameters and ε is a vector of independent random variables (noise) symmetrically distributed with mean zero and variance σ^2. The estimation of β can be obtained under several different optimality criteria. For β unrestricted, the classical least squares estimator, $\tilde{\beta}$, where

$$\tilde{\beta} = (X^TX)^{-1} X^TY \ , \tag{2}$$

has the smallest variance among the class of unbiased linear functions of y. However, the least squares estimator is extremely sensitive to large values of $|\varepsilon|$, outliers, particularly when the sample size, n, is small relative to p, say $n \leq 2(p+1)$. In addition, the least squares estimator does not have the flexibility of allowing restrictions to be placed on β. These two drawbacks suggest that an optimality criteria other than least squares be considered. Several authors (Barrodale (1968), Charnes and Cooper (1964), Gentle, Kennedy and Sposito (1977), Harris (1950), Harter (1974), Rice and White (1964), Taylor (1973) have suggested that

$$\sum_{i=1}^{n} | y_i - X_i\beta| \tag{3}$$

should be minimized with respect to β where y_i is the i-th observation and X_i is the i-th row of X. The estimator, $\hat{\beta}$, which minimizes the sum of the absolute residuals is often called the L_1 estimator.

The L_1 estimate is not necessarily unique since it depends upon the

method of computation. However, conventional linear programming algorithms can calculate the L_1 estimate with m added linear restrictions on β in the forms,

$$A\beta \leq b \quad \text{or} \tag{4}$$

$$A\beta \leq b \quad \text{or}$$

$$A\beta = b$$

where A is an m x p matrix of known coefficients for β and b is a vector of m constants.

The computer program MR. A implements an algorithm for Minimizing the sum of the Absolute Residuals where β is restricted by any of the relationships given in (4). The algorithm uses the least squares estimator $\tilde{\beta}$ as an initial unbiased antisymmetrical estimator of β where

$$\beta - \tilde{\beta}\ (\varepsilon) = -\ [\beta - \tilde{\beta}\ (-\varepsilon)] \tag{5}$$

in order to provide efficiency in computation as suggested by Barrodale and Roberts (1973).

The estimation of the unrestricted L_1 estimator is only a first step in the completion of the task of estimating β in model (1) subject to restrictions in (4). The second step is the estimation of the covariances of $\hat{\beta}$. This estimation would usually be required for any further developments in hypothesis testing or confidence intervals. MR. A contains a mini-Monte Carlo procedure similar to that used in the algorithm MRS. A (Book, Booker, Hartley, Sielken 1980) for estimating the covariance of the restricted L_1 estimator. This feature sets MR. A apart from other restricted L_1 procedures.

2. Computational Procedure

The problem of minimizing the sum of the absolute residuals can be formulated as follows:

$$\min \sum_{i=1}^{n} (r_{1i} + r_{2i}) \tag{6}$$

subject to

$$
\begin{aligned}
y_i - X_i\beta &= (r_{1i} - r_{2i}), \quad i = 1, \ldots, n, \\
r_{1i}, r_{2i} &\geq 0, \qquad\qquad (7) \\
A_i\beta &\left\{\begin{matrix} \leq \\ \geq \\ = \end{matrix}\right\} b_i,
\end{aligned}
$$

where the i-th residual, r_i, is expressed as the difference between two positive components, r_{1i} and r_{2i}. To promote the unbiasedness of the resulting L_1 estimator and to increase the efficiency of computation in the algorithm, the problem is reformulated following Hartley and Sielken (1973) by introducing the antisymmetrical least squares estimator, $\beta_0 = \hat{\beta}$, and transforming (7) to

$$
\begin{aligned}
y_i - X_i\beta_0 - X_i\,(\beta - \beta_0) &= r_{1i} - r_{2i}, \quad i = 1, \ldots, n \qquad (8) \\
A_i(\beta - \beta_0) &\left\{\begin{matrix} \leq \\ \geq \\ = \end{matrix}\right\} b_i - A_i\beta_0
\end{aligned}
$$

Then, using

$$
\begin{aligned}
\beta &= \beta^{(1)} - \beta^{(2)}, \\
\beta_0 &= \beta_0^{(1)} - \beta_0^{(2)}
\end{aligned}
$$

with $\beta^{(1)}, \beta^{(2)}, \beta_0^{(1)}, \beta_0^{(2)} \geq 0$ in (8) yields

$$
\begin{aligned}
X_i\,(\beta^{(1)} + \beta_0^{(2)}) - X_i\,(\beta^{(2)} + \beta_0^{(1)}) + r_{1i} - r_{2i} &= y_i - X_i\beta_0 \\
A_i\,(\beta^{(1)} + \beta_0^{(2)}) - A_i\,(\beta^{(2)} + \beta_0^{(1)}) &\left\{\begin{matrix} \leq \\ \geq \\ = \end{matrix}\right\} b_i - A_i\beta_0 \qquad (9)
\end{aligned}
$$

or equivalently

$$
\begin{aligned}
X_iB_1 - X_iB_2 + r_{1i} - r_{2i} &= y_i - X_i\beta_0 \\
A_iB_1 - A_iB_2 &\left\{\begin{matrix} \leq \\ \geq \\ = \end{matrix}\right\} b_i - A_i\beta_0
\end{aligned}
$$

for $i = 1, \ldots, n$ where

$$B_1 = \beta^{(1)} + \beta_0^{(2)} \geq 0 \tag{10}$$

$$B_2 = \beta^{(2)} + \beta_0^{(1)} \geq 0 \; .$$

Now, in order to promote unbiasedness of $\hat{\beta}$ and to partially compensate for any idiosyncrasies in the particular linear programming algorithm used to solve the problem in (6), (7), (9), (10), the problem is considered in two equivalent forms P_1 and P_2 with MR. A randomly selecting either P_1 or P_2 with probability 1/2. In matrix notation, the problems P_1 and P_2 are as follows:

$$P_1: \quad \min \sum_{i=1}^{n} (r_{1i} + r_{2i})$$

subject to

$$\begin{bmatrix} X & -X & 1 & -I \\ A & -A & 0 & 0 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ r_1 \\ r_2 \end{bmatrix} \begin{matrix} = \\ \left\{ \begin{matrix} \leq \\ \geq \\ = \end{matrix} \right\} \end{matrix} \begin{bmatrix} y - X\beta_0 \\ b - A\beta_0 \end{bmatrix}$$

$$B_1 , B_2 , r_1 , r_2 \geq 0;$$

$$P_2: \quad \min \sum_{i=1}^{n} (r_{1i} + r_{2i})$$

subject to

$$\begin{bmatrix} -X & X & I & -I \\ -A & A & 0 & 0 \end{bmatrix} \begin{bmatrix} B_2 \\ B_1 \\ r_1 \\ r_2 \end{bmatrix} \begin{matrix} = \\ \left\{ \begin{matrix} \leq \\ \geq \\ = \end{matrix} \right\} \end{matrix} \begin{bmatrix} y - X\beta_0 \\ b - A\beta_0 \end{bmatrix}$$

$$B_1 \, , \, B_2 \, , \, r_1 \, , \, r_2 \;\geq\; 0.$$

After MR. A selects and solves either P_1 or P_2, the L_1 estimator is $\hat{\beta}$ where

$$\hat{\beta} = B_1 - B_2 + \beta_0. \tag{11}$$

In cases where the sample size is large, say $n >> (p+1)^2$, the sample could be randomly divided into G groups, then $\hat{\beta}_g$ estimated for each group g separately, and the covariance of $\hat{\beta}$ estimated from the sample covariance of the $\hat{\beta}_g$'s.

MR. A estimates the covariance of $\hat{\beta}$ for any sample size using a mini-Monte Carlo procedure. A Monte-Carlo estimate is obtained by essentially generating several sets of n y's, finding the L_1 estimate for each set, and computing the sample covariance. There are two difficulties with generating the y's; namely,

(1) β is unknown, and

(2) σ^2 is unknown.

To overcome the first difficulty, the sum of the absolute residuals is expressed as

$$\begin{aligned}\sum_{i=1}^{n} | y_i - X_i\hat{\beta}| &= \sum_{i=1}^{n} | X_i\beta + \varepsilon_i - X_i\hat{\beta} | \\ &= \sum_{i=1}^{n} | \varepsilon_i - X_i(\hat{\beta}-\beta)| \\ &= \sum_{i=1}^{n} | \varepsilon_i - X_i \, \delta\beta |\end{aligned} \tag{12}$$

using (1) and $\delta\beta = \hat{\beta}-\beta$. Note that the covariance of the L_1 estimator of $\delta\beta$ is the same as the covariance of $\hat{\beta}$. Thus, only sets of n ε's need be generated. To deal with the second difficulty (the unknown σ^2), note that

$$\sum_{i=1}^{n} | \varepsilon_i - X_i \delta\beta | = \sigma \sum_{i=1}^{n} | \varepsilon_i^* - X_i \delta\beta^* | \qquad (13)$$

and the restrictions on β in (4) become

$$A\hat{\beta} - A\beta \left\{ \begin{matrix} \leq \\ \geq \\ = \end{matrix} \right\} b - A\beta$$

$$A(\delta\beta) \left\{ \begin{matrix} \leq \\ \geq \\ = \end{matrix} \right\} b - A\beta$$

$$A(\delta\beta^*) \left\{ \begin{matrix} \leq \\ \geq \\ = \end{matrix} \right\} \frac{b - A\beta}{\sigma} \qquad (14)$$

where $\delta\beta^* = (\hat{\beta}-\beta)/\sigma$ and where $\varepsilon_i^* = \varepsilon_i/\sigma$ and is symmetrically distributed with mean 0 and variance 1. Furthermore,

$$\text{Covariance } (\hat{\beta}) = \text{Covariance } (\hat{\delta\beta}) = \sigma^2 \text{ Covariance } (\hat{\delta\beta}^*). \qquad (15)$$

The mini-Monte Carlo procedure generates K sets of n ε^*'s; finds the K L_1 estimates $\hat{\delta\beta}^*$; and estimates the covariance of $\hat{\beta}$ by $\hat{\sigma}$ times the sample covariance of $\hat{\delta\beta}^*$ where $\hat{\sigma}$ is calculated from

$$\hat{\sigma} = \left\{ \frac{\sum_{i=1}^{n} | y_i - X_i\hat{\beta} | /(n-p)}{\frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{n} | \varepsilon_{ik}^* - X_i \hat{\delta\beta}_k^* | /(n-p)} \right\}^{1/2} \qquad (16)$$

This estimator of σ reflects both the variability in $y_i - X_i\beta$ and the variability associated with the linear programming algorithm.

In the estimation of each $\delta\beta_k^*$, an additional problem arises from the right hand side of (14) being a function of β and σ. This problem is overcome by estimating β using (11) and estimating σ using (16) for each sample, $k \geq 2$, as it is generated. For the first sample ($k = 1$), σ is estimated by the numerator of (16). This numerator has been shown to provide a reasonable alternative estimator of σ [Book, Booker, Hartley,

MR. A allows the user to generate the ε^*'s from the uniform, normal or double exponential distributions. These distributions were selected in order to represent short, medium and long tailed distributions, respectively. These three distributions are also interesting because maximum likelihood estimation corresponds to minimizing the maximum absolute residual if the ε's are uniform, minimizing the sum of squared residuals if the ε's are normal, and minimizing the sum of the absolute residuals if the ε's are double exponential.

MR. A also allows the user to assign weight coefficients to the residuals, changing (6) to

$$\min \sum_{i=1}^{n} w_i(r_{1i} + r_{2i})$$

where w_i is the weight coefficient for the i-th residual, r_i. If the w_i's are not all equal to one, then the objective function in the mini-Monte Carlo study becomes

$$\sum_{i=1}^{n} w_i \mid \varepsilon_i^* - X_i\delta\beta^* \mid$$

and the estimate of σ also reflects the w_i, namely,

$$\hat{\sigma} = \frac{\sum_{i=1}^{n} w_i \mid y_i - X_i\hat{\beta} \mid /(n-p)}{\frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{n} w_i \mid \varepsilon_{ik}^* - X_i(\hat{\delta\beta^*})_k \mid /(n-p)} .$$

3. MR. A Users' Guide and Sample Problem

MR. A consists of a main program and subroutines. The functions of these components are given in Table 1 at the end of this section.

The input to the computer program consists of five basic card groups. Input instructions are also briefly documented in the program comment statements.

The first card or card image contains the ancillary statistics required for the specific problem as follows:

First Card:

Card Column	Variable Name	Description
1-3	IP	= p = number of beta parameters (format I3; i.e., a 3 digit integer, right justified)
4-6	NOBS	= n = number of observations (format I3)
7-9	NC	= m = number of constraints (format I3)
11-20	NSEED	Ten digit random number $\leq$ 2147483647 (format I10)
21-25	ISAM	= K = number of samples for the mini-Monte Carlo study (format I5)
27	IWRIT0	= 1 if the problem input is to be printed = 0 otherwise
29	IWRIT1	= 1 if the main results of the L_1 estimation are to be printed = 0 otherwise
31	IWRIT2	= 1 if the intermediate results of the L_1 estimation are to be printed = 0 otherwise
33	IWRIT3	= 1 if the basis inverse is to be printed = 0 otherwise
35	IWRIT4	= 1 if the main results of the mini-Monte Carlo study are to be printed = 0 otherwise
37	IWRIT5	= 1 if the intermediate results of the mini-Monte Carlo study are to be printed = 0 otherwise

Card Column	Variable Name	Description
39	IWRIT6	= 1 if the intermediate results in the determination of the covariance of $\hat{\beta}$ are to be printed = 0 otherwise
41	IOPTN	= 1 if the ε^*'s are to be normally distributed = 2 if the ε^*'s are to be double exponentially distributed = 3 if the ε^*'s are to be uniformly distributed.
43	IWT	= 1 if the residuals are assigned weights, w_i = 0 if the residuals are not weighted.

The remaining card input instructions are as follows:

Card Type	Variable Name	Description
Second card group:	$ITYPE_i$, i=1, ..., NC	The type of the i-th constraint on β = 0 if the constraint is $\leq$ type = 1 if the constraint is $\geq$ type = 2 if the constraint is = type (Format I2)
Third card group:	y_i, i=1, ..., NOBS	The observations [format (10F8.3); i.e., ten eight digit numbers with either a decimal point included or last five digits are assumed to be to the right of a supplied decimal point]

Card Type	Variable Name	Description
	x_{ij}, i=1, ..., NOBS j=1, ..., IP	The matrix of beta coefficients, read in by rows [format (10F8.3)].
Fourth card group:	A_{ij}, i=1, ..., NC j=1, ..., IP	The matrix of beta coefficients in the constraints on beta, read in by rows [format (10F8.3)].
	b_i, i=1, ..., NC	The right-hand sides of the beta constraints [format (10F8.3)].
Fifth card group	w_i, i=1, ..., NOBS	The weights assigned to the residuals [format (10F8.3)]. (Included only if IWT = 1).

The user then supplies a title card of length 80 spaces or less following the fifth card group.

The size of the problem which MR. A can solve is limited only by the current dimension statements. These restrict the number of observations to 20 ($n \leq 20$), the number of parameters to 10 ($p \leq 10$), the number of constraints to 10 ($m \leq 10$), and the number of samples in the mini-Monte Carlo study to 100 ($K \leq 100$). However, expansion can easily be accomplished by increasing these dimensions in the dimension statements as documented in the program.

MR. A is written in Fortran IV language and is compatible with Fortran G and H and WATFIV language compilers. The program uses double precision arithmatic.

MR. A has been tested on several problems on an AMDAHL 470 V6 and should be compatible with all IBM compilers. MR. A will execute small problems such as $n = 5$, $p = 3$, $K = 6$, $m = 3$ in less than two seconds. Problems of the size $n = 20$, $p = 2$, $K = 30$, $m = 3$ take up to a minute of execution time. A sample problem with sample input and output is given

in Appendix A and B respectively. The program listing of MR. A is given in Appendix C.

Table 1

Components of MR. A and their Functions

Component	Function
MAIN	Reads data and generates output.
	Performs L_1 estimation of $\hat{\beta}$.
	Determines $\hat{\sigma}$ and the estimated covariance of $\hat{\beta}$.
DUAL	Carries out the mini-Monte Carlo study.
MCRHS	Forms the right hand sides of (14) for the mini-Monte Carlo study.
SELECT	Selects P_1 or P_2 for solution.
RAND	Generates random uniform variables ranging from 0 to 1.
RHANDS	Forms the right-hand sides of (9).
BTILDE	Constructs the least squares estimate of β; i.e., β_0.
INVERT	Inverts an n x n matrix.
NORMAL	Generates a vector of normally distributed random variables with mean 0 and variance 1 for the mini-Monte Carlo study.
DOUBLE	Generates a vector of double exponentially distributed random variables with mean 0 and variance 1 for the mini-Monte Carlo study.
UNIFRM	Generates a vector of uniformly distributed random variables with mean 0 and variance 1 for the mini-Monte Carlo study.

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APPENDIX A. SAMPLE INPUTS

```
  2  5  3 1872539680   20 0 0 0 0 0 0 0 1 0
 2 1 0
   4.257  -1.983   0.024  -3.180  -3.986
   1.000   1.120
   1.000   1.950
   1.000   3.020
   1.000   5.430
   1.000   6.590
   1.000   3.000  -1.000
THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH NO OPTIONAL PRINTOUTS.
```

```
  2  5  3 1872539680   20 1 1 1 1 1 1 1 1 0
 2 1 0
   4.257  -1.983   0.024  -3.180  -3.986
   1.000   1.120
   1.000   1.950
   1.000   3.020
   1.000   5.430
   1.000   6.590
   1.000   3.000  -1.000
THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH ALL OPTIONAL PRINTOUTS.
```

APPENDIX B. SAMPLE OUTPUTS

```
          MR. A. :

MINIMIZES THE ABSOLUTE RESIDUALS.
```

(Sample output with no optional printouts)

```
     THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING
THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.
IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN
ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED
REGRESSION PARAMETERS.
     IN ADDITION, LINEAR CONSTRAINTS MAY
BE PLACED ON THE PARAMETER VECTOR, BETA.
THESE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONTSTRAINTS.
     THE FOLLOWING PROCEDURE DEVELOPED BY :
         D.N. BOOK
         J.B. BOOKER
         H.O. HARTLEY
         R.L. SIELKEN, JR.
     INSTITUTE OF STATISTICS
     TEXAS A & M UNIVERSTIY
     COLLEGE STATION, TEXAS 77843

     INQUIRIES AND COMMENTS SHOULD BE ADDRESSED TO:
         ROBERT L. SIELKEN, JR.

     THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY ACKNOWLEDGED.
THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH NO OPTIONAL PRINTOUTS.

NUMBER OF OBSERVATIONS =      5
NUMBER OF PARAMETERS =     2
USER SUPPLIED RANDOM INTEGER, NSEED =  1872539680
SAMPLE SIZE FOR THE MINI MONTE CARLO STUDY =    20
NUMBER OF BETA CONSTRAINTS =     3
    NUMBER OF < OR = BETA CONSTRAINTS =     1
    NUMBER OF > OR = BETA CONSTRAINTS =     1
    NUMBER OF = BETA CONSTRAINTS =     1
THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0, OF THE REGRESSION PARAMETER VECTOR, BETA
       LEAST SQUARES ESTIMATE OF BETA(  1) =       3.227002
       LEAST SQUARES ESTIMATE OF BETA(  2) =      -1.159747
MR.A'S ANSWER :            THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS
       L1 ESTIMATE OF BETA(  1) =       3.000000
       L1 ESTIMATE OF BETA(  2) =      -2.000000
THE RESIDUALS :
       THE   1-TH RESIDUAL =       3.497000
       THE   2-TH RESIDUAL =      -1.083000
       THE   3-TH RESIDUAL =       3.064000
       THE   4-TH RESIDUAL =       4.680000
       THE   5-TH RESIDUAL =       6.194000
THE SUM OF THE ABSOLUTE RESIDUALS =      18.518000
THE MAXIMUM ABSOLUTE RESIDUAL =      6.194000




MAIN RESULTS OF THE MINI MONTE CARLO STUDY

ESTIMATED VALUE OF SIGMA =     2.817978
ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER VECTOR, BETA
            3.565439    -0.970588
           -0.970588     0.435623
```

MR. A. :

(Sample output with optional printouts)

```
MINIMIZES THE ABSOLUTE RESIDUALS.

     THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING
THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.
IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN
ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED
REGRESSION PARAMETERS.
     IN ADDITION, LINEAR CONSTRAINTS MAY
BE PLACED ON THE PARAMETER VECTOR, BETA.
THESE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONTSTRAINTS.
     THE FOLLOWING PROCEDURE DEVELOPED BY :
         D.N. BOOK
         J.B. BOOKER
         H.O. HARTLEY
         R.L. SIELKEN, JR.
     INSTITUTE OF STATISTICS
     TEXAS A & M UNIVERSTIY
     COLLEGE STATION, TEXAS 77843

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THIS IS AN EXAMPLE PROBLEM OF MR. A. WITH ALL OPTIONAL PRINTOUTS.

NUMBER OF OBSERVATIONS =     5
NUMBER OF PARAMETERS =     2
USER SUPPLIED RANDOM INTEGER, NSEED =  1872539680
SAMPLE SIZE FOR THE MINI MONTE CARLO STUDY =    20
NUMBER OF BETA CONSTRAINTS =     3
    NUMBER OF < OR = BETA CONSTRAINTS =      1
    NUMBER OF > OR = BETA CONSTRAINTS =      1
    NUMBER OF = BETA CONSTRAINTS =      1
THE LINEAR REGRESSION IS Y = X*BETA + EPSILON
WHERE
Y CONTAINS THE OBSERVATIONS,
BETA IS A VECTOR CONTAINING THE REGRESSION PARAMETERS,
THE I-TH ROW OF X CONTAINS THE COEFFICIENTS OF BETA
CORRESPONDING TO THE I-TH OBSERVATION, AND
EPSILON IS A RANDOM VARIABLE WITH MEAN ZERO AND VARIANCE
SIGMA-SQUARED REPRESENTING RANDOM VARIABILITY FROM
X*BETA.
     THE Y VECTOR
                  4.25700
                 -1.98300
                  0.02400
                 -3.18000
                 -3.98600
     THE X MATRIX
             1.0    1.1
             1.0    2.0
             1.0    3.0
             1.0    5.4
             1.0    6.6
 THE CONSTRAINTS PLACED ON BETA OF THE FORM A*BETA = B
   WHERE THE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONSTRAINTS.
   A = THE MATRIX OF COEFFICIENTS WHOSE I-TH ROW CORRESPONDS TO THE I-TH INEQUALITY.
     THE RIGHT HAND SIDES OF THE BETA CONSTRAINTS, THE VECTOR B
            1.00000
            3.00000
           -1.00000
     THE COEFFICIENTS FOR THE BETA CONSTRAINTS, THE A MATRIX
        1.0000    1.0000
        1.0000    0.0000
        0.0000    1.0000
     THE INVERSE OF XTX
          0.810298      -0.168497
         -0.168497       0.046521
     THE VALUE OF BETA0 TO COMPUTE RHS
          3.227002      -1.159747
     THE RIGHT HAND SIDES, Y-X*BETA0
         2.32891    -2.94850    0.29943    -0.10958    0.42973
     THE RIGHT HAND SIDES, B-A*BETA0
        -1.06726    -0.22700    0.15975
     THE PROBLEM P2 HAS BEEN SELECTED
THE LINEAR PROGRAMMING PROBLEM AS IT WAS CREATED
     THE OBJECTIVE FUNCTION COEFFICIENTS
          C(  1) =      0.00000
          C(  2) =      0.00000
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        C(  3) =       0.00000
        C(  4) =       0.00000
        C(  5) =      -1.00000
        C(  6) =      -1.00000
        C(  7) =      -1.00000
        C(  8) =      -1.00000
        C(  9) =      -1.00000
        C( 10) =      -1.00000
        C( 11) =      -1.00000
        C( 12) =      -1.00000
        C( 13) =      -1.00000
        C( 14) =      -1.00000
        C( 15) =       0.00000
        C( 16) =       0.00000
    THE RIGHT-HAND SIDES
        BRHS(  1) =       2.32891
        BRHS(  2) =      -2.94850
        BRHS(  3) =       0.29943
        BRHS(  4) =      -0.10958
        BRHS(  5) =       0.42973
        BRHS(  6) =      -1.06726
        BRHS(  7) =      -0.22700
        BRHS(  8) =       0.15975
    THE CONSTRAINT MATRIX A
                -0.00000      -0.00000      -0.00000      -0.00000       1.00000
   1.00000       1.00000       1.00000       1.00000       1.00000
   1.00000       1.00000       1.00000       1.00000      -0.00000
  -0.00000
                 0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000
                -1.00000      -1.12000       1.00000       1.12000       1.00000
   0.00000       0.00000       0.00000       0.00000      -1.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000
                -1.00000      -1.95000       1.00000       1.95000       0.00000
   1.00000       0.00000       0.00000       0.00000       0.00000
  -1.00000       0.00000       0.00000       0.00000       0.00000
   0.00000
                -1.00000      -3.02000       1.00000       3.02000       0.00000
   0.00000       1.00000       0.00000       0.00000       0.00000
   0.00000      -1.00000       0.00000       0.00000       0.00000
   0.00000
                -1.00000      -5.43000       1.00000       5.43000       0.00000
   0.00000       0.00000       1.00000       0.00000       0.00000
   0.00000       0.00000      -1.00000       0.00000       0.00000
   0.00000
                -1.00000      -6.59000       1.00000       6.59000       0.00000
   0.00000       0.00000       0.00000       1.00000       0.00000
   0.00000       0.00000       0.00000      -1.00000       0.00000
   0.00000
                -1.00000      -1.00000       1.00000       1.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000
                -1.00000      -0.00000       1.00000       0.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000       0.00000       0.00000       0.00000      -1.00000
   0.00000
                -0.00000      -1.00000       0.00000       1.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   0.00000       0.00000       0.00000       0.00000       0.00000
   1.00000
     THE INITIAL BASIS INVERSE
    THE 1 -TH ROW:   0.10000000D 01   0.00000000D 00  -0.10000000D 01   0.10000000D 01  -0.10000000D 01
 0.10000000D 01  -0.10000000D 01   0.00000000D 00   0.00000000D 00   0.00000000D 00
    THE 2 -TH ROW:   0.00000000D 00   0.10000000D 01   0.00000000D 00   0.00000000D 00   0.00000000D 00
 0.00000000D 00   0.00000000D 00   0.10000000D 01   0.00000000D 00   0.00000000D 00
    THE 3 -TH ROW:   0.00000000D 00   0.00000000D 00   0.10000000D 01   0.00000000D 00   0.00000000D 00
 0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
    THE 4 -TH ROW:   0.00000000D 00   0.00000000D 00   0.00000000D 00  -0.10000000D 01   0.00000000D 00
 0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
    THE 5 -TH ROW:   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.10000000D 01
 0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
    THE 6 -TH ROW:   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
-0.10000000D 01   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
    THE 7 -TH ROW:   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
 0.00000000D 00   0.10000000D 01   0.00000000D 00   0.00000000D 00   0.00000000D 00
    THE 8 -TH ROW:   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00   0.00000000D 00
 0.00000000D 00   0.00000000D 00  -0.10000000D 01   0.00000000D 00   0.00000000D 00
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      THE 9 -TH ROW:     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00
 0.00000000D 00     0.00000000D 00     0.00000000D 00    -0.10000000D 01     0.00000000D 00
      THE10 -TH ROW:     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00
 0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.10000000D 01
       THE VARIABLES INITIALLY IN THE AUGMENTED BASIS MATRIX
            0,50, 5,11, 7,13, 9,51,16,15,
       THE INITIAL VALUES OF THE BASIC VARIABLES
            XB(  1) =     -0.61161D 01
            XB(  2) =     -0.10673D 01
            XB(  3) =      0.23289D 01
            XB(  4) =      0.29485D 01
            XB(  5) =      0.29943D 00
            XB(  6) =      0.10958D 00
            XB(  7) =      0.42973D 00
            XB(  8) =      0.10673D 01
            XB(  9) =      0.22700D 00
            XB( 10) =      0.15975D 00
       THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =    -0.1067255616D 01
       PHASE 1 NOW BEGINS.
                          PHASE 1 :  ITERATION   1
       THE REDUCED COSTS
            THE   1-TH REDUCED COST =     0.1000000000D 01
            THE   2-TH REDUCED COST =     0.1000000000D 01
            THE   3-TH REDUCED COST =    -0.1000000000D 01
            THE   4-TH REDUCED COST =    -0.1000000000D 01
            THE   5-TH REDUCED COST =    -0.0000000000D 00
            THE   6-TH REDUCED COST =    -0.0000000000D 00
            THE   7-TH REDUCED COST =    -0.0000000000D 00
            THE   8-TH REDUCED COST =    -0.0000000000D 00
            THE   9-TH REDUCED COST =    -0.0000000000D 00
            THE  10-TH REDUCED COST =    -0.0000000000D 00
            THE  11-TH REDUCED COST =    -0.0000000000D 00
            THE  12-TH REDUCED COST =    -0.0000000000D 00
            THE  13-TH REDUCED COST =    -0.0000000000D 00
            THE  14-TH REDUCED COST =    -0.0000000000D 00
            THE  15-TH REDUCED COST =    -0.0000000000D 00
            THE  16-TH REDUCED COST =    -0.0000000000D 00
       X(  1) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX
       WHICH HAS THE LARGEST POSITIVE REDUCED COST.
       THEREFORE X(  1) WILL ENTER THE BASIS.
       THE COMPONENTS OF Y(-,  1)
            Y(  3,  1) =    -0.1000000000D 01
            Y(  4,  1) =     0.1000000000D 01
            Y(  5,  1) =    -0.1000000000D 01
            Y(  6,  1) =     0.1000000000D 01
            Y(  7,  1) =    -0.1000000000D 01
            Y(  8,  1) =     0.1000000000D 01
            Y(  9,  1) =     0.1000000000D 01
            Y( 10,  1) =     0.0000000000D 00
       THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.
       THE XB(I)/Y(I,  1) RATIOS WITH Y(I,  1) > 0. ARE:
                 0.29485D 01= XB(  4)/Y(  4,  1)
                 0.10958D 00= XB(  6)/Y(  6,  1)
                 0.10673D 01= XB(  8)/Y(  8,  1)
                 0.22700D 00= XB(  9)/Y(  9,  1)
       THE MINIMUM RATIO IS XB(  6)/Y(  6,  1) =     0.1095781257D 00.
       THEREFORE THE   6-TH VARIABLE LEAVES THE BASIS.
       THE BASIC VARIABLES ARE NOW
            0,50, 5,11, 7, 1, 9,51,16,15,
       THE VALUES OF THE BASIC VARIABLES ARE NOW
            XB(  1) = X(  0) =    -0.6225727047D 01
            XB(  2) = X( 50) =    -0.9576774908D 00
            XB(  3) = X(  5) =     0.2438492103D 01
            XB(  4) = X( 11) =     0.2838918209D 01
            XB(  5) = X(  7) =     0.4090106653D 00
            XB(  6) = X(  1) =     0.1095781257D 00
            XB(  7) = X(  9) =     0.5393060698D 00
            XB(  8) = X( 51) =     0.9576774908D 00
            XB(  9) = X( 16) =     0.1174241027D 00
            XB( 10) = X( 15) =     0.1597466119D 00
       THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.95768D 00
       THE CURRENT BASIS INVERSE IS AS FOLLOWS:
         THE  1-TH ROW:     0.10000000D 01    -0.00000000D 00    -0.10000000D 01     0.10000000D 01    -0.10000000D 01
 0.20000000D 01    -0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
         THE  2-TH ROW:     0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.10000000D 01    -0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00
         THE  3-TH ROW:     0.00000000D 00    -0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00
-0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
         THE  4-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.10000000D 01    -0.00000000D 00
 0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
         THE  5-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01
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-0.10000000D 01     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
          THE  6-TH ROW:     0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00
-0.10000000D 01      0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00
          THE  7-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.10000000D 01      0.10000000D 01      0.00000000D 00     -0.00000000D 00     -0.00000000D 00
          THE  8-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
 0.10000000D 01     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00     -0.00000000D 00
          THE  9-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
 0.10000000D 01     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
          THE 10-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00      0.10000000D 01
                      PHASE 1 :  ITERATION   2
        THE REDUCED COSTS
            THE   1-TH REDUCED COST =    -0.0000000000D 00
            THE   2-TH REDUCED COST =    -0.4430000000D 01
            THE   3-TH REDUCED COST =    -0.0000000000D 00
            THE   4-TH REDUCED COST =     0.4430000000D 01
            THE   5-TH REDUCED COST =    -0.0000000000D 00
            THE   6-TH REDUCED COST =    -0.0000000000D 00
            THE   7-TH REDUCED COST =    -0.0000000000D 00
            THE   8-TH REDUCED COST =     0.1000000000D 01
            THE   9-TH REDUCED COST =    -0.0000000000D 00
            THE  10-TH REDUCED COST =    -0.0000000000D 00
            THE  11-TH REDUCED COST =    -0.0000000000D 00
            THE  12-TH REDUCED COST =    -0.0000000000D 00
            THE  13-TH REDUCED COST =    -0.1000000000D 01
            THE  14-TH REDUCED COST =    -0.0000000000D 00
            THE  15-TH REDUCED COST =    -0.0000000000D 00
            THE  16-TH REDUCED COST =    -0.0000000000D 00
        X(  4) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX
        WHICH HAS THE LARGEST POSITIVE REDUCED COST.
        THEREFORE X(  4) WILL ENTER THE BASIS.
        THE COMPONENTS OF Y(-,  4)
            Y(  3,  4) =    -0.4310000000D 01
            Y(  4,  4) =     0.3480000000D 01
            Y(  5,  4) =    -0.2410000000D 01
            Y(  6,  4) =    -0.5430000000D 01
            Y(  7,  4) =     0.1160000000D 01
            Y(  8,  4) =     0.4430000000D 01
            Y(  9,  4) =     0.5430000000D 01
            Y( 10,  4) =     0.1000000000D 01
        THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.
        THE XB(I)/Y(I,  4) RATIOS WITH Y(I,  4) > 0. ARE:
                0.81578D 00= XB(  4)/Y(  4,  4)
                0.46492D 00= XB(  7)/Y(  7,  4)
                0.21618D 00= XB(  8)/Y(  8,  4)
                0.21625D-01= XB(  9)/Y(  9,  4)
                0.15975D 00= XB( 10)/Y( 10,  4)
        THE MINIMUM RATIO IS XB(  9)/Y(  9,  4) =      0.2162506495D-01.
        THEREFORE THE   9-TH VARIABLE LEAVES THE BASIS.
        THE BASIC VARIABLES ARE NOW
             0,50, 5,11, 7, 1, 9,51, 4,15,
        THE VALUES OF THE BASIC VARIABLES ARE NOW
            XB(  1) = X(  0) =    -0.6270707182D 01
            XB(  2) = X( 50) =    -0.8618784530D 00
            XB(  3) = X(  5) =     0.2531696133D 01
            XB(  4) = X( 11) =     0.2763662983D 01
            XB(  5) = X(  7) =     0.4611270718D 00
            XB(  6) = X(  1) =     0.2270022283D 00
            XB(  7) = X(  9) =     0.5142209945D 00
            XB(  8) = X( 51) =     0.8618784530D 00
            XB(  9) = X(  4) =     0.2162506495D-01
            XB( 10) = X( 15) =     0.1381215470D 00
        THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =     -0.86188D 00
        THE CURRENT BASIS INVERSE IS AS FOLLOWS:
          THE  1-TH ROW:     0.10000000D 01     -0.00000000D 00     -0.10000000D 01      0.10000000D 01     -0.10000000D 01
 0.16169429D 01     -0.10000000D 01     -0.00000000D 00      0.38305709D 00     -0.00000000D 00
          THE  2-TH ROW:     0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.18416206D 00     -0.00000000D 00      0.10000000D 01     -0.81583794D 00     -0.00000000D 00
          THE  3-TH ROW:     0.00000000D 00     -0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.00000000D 00
-0.20626151D 00     -0.00000000D 00     -0.00000000D 00     -0.79373849D 00     -0.00000000D 00
          THE  4-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
 0.35911602D 00     -0.00000000D 00     -0.00000000D 00      0.64088398D 00     -0.00000000D 00
          THE  5-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00      0.10000000D 01
-0.55616943D 00     -0.00000000D 00     -0.00000000D 00     -0.44383057D 00     -0.00000000D 00
          THE  6-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.69388939D-16     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
          THE  7-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.12113628D 01      0.10000000D 01     -0.00000000D 00      0.21362799D 00     -0.00000000D 00
          THE  8-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
 0.18416206D 00     -0.00000000D 00     -0.10000000D 01      0.81583794D 00     -0.00000000D 00
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        THE  9-TH ROW:     0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00
 0.18416206D 00      0.00000000D 00      0.00000000D 00     -0.18416206D 00      0.00000000D 00
        THE 10-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.18416206D 00     -0.00000000D 00     -0.00000000D 00      0.18416206D 00      0.10000000D 01
                    PHASE 1 :  ITERATION   3
      THE REDUCED COSTS
           THE   1-TH REDUCED COST =    -0.0000000000D 00
           THE   2-TH REDUCED COST =    -0.2220446049D-15
           THE   3-TH REDUCED COST =    -0.0000000000D 00
           THE   4-TH REDUCED COST =     0.2220446049D-15
           THE   5-TH REDUCED COST =    -0.0000000000D 00
           THE   6-TH REDUCED COST =    -0.0000000000D 00
           THE   7-TH REDUCED COST =    -0.0000000000D 00
           THE   8-TH REDUCED COST =     0.1841620626D 00
           THE   9-TH REDUCED COST =    -0.0000000000D 00
           THE  10-TH REDUCED COST =    -0.0000000000D 00
           THE  11-TH REDUCED COST =    -0.0000000000D 00
           THE  12-TH REDUCED COST =    -0.0000000000D 00
           THE  13-TH REDUCED COST =    -0.1841620626D 00
           THE  14-TH REDUCED COST =    -0.0000000000D 00
           THE  15-TH REDUCED COST =    -0.8158379374D 00
           THE  16-TH REDUCED COST =    -0.0000000000D 00
      X(  8) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX
      WHICH HAS THE LARGEST POSITIVE REDUCED COST.
      THEREFORE X(  8) WILL ENTER THE BASIS.
      THE COMPONENTS OF Y(-,  8)
           Y(  3,  8) =    -0.2062615101D 00
           Y(  4,  8) =     0.3591160221D 00
           Y(  5,  8) =    -0.5561694291D 00
           Y(  6,  8) =    -0.6938893904D-16
           Y(  7,  8) =    -0.1213627993D 01
           Y(  8,  8) =     0.1841620626D 00
           Y(  9,  8) =     0.1841620626D 00
           Y( 10,  8) =    -0.1841620626D 00
      THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.
      THE XB(I)/Y(I,  8) RATIOS WITH Y(I,  8) > 0. ARE:
              0.76957D 01= XB(  4)/Y(  4,  8)
              0.46800D 01= XB(  8)/Y(  8,  8)
              0.11742D 00= XB(  9)/Y(  9,  8)
      THE MINIMUM RATIO IS XB(  9)/Y(  9,  8) =     0.1174241027D 00.
      THEREFORE THE   9-TH VARIABLE LEAVES THE BASIS.
      THE BASIC VARIABLES ARE NOW
           0,50, 5,11, 7, 1, 9,51, 8,15,
      THE VALUES OF THE BASIC VARIABLES ARE NOW
           XB(  1) = X(  0) =     -0.6577999355D 01
           XB(  2) = X( 50) =     -0.8402533881D 00
           XB(  3) = X(  5) =      0.2555916205D 01
           XB(  4) = X( 11) =      0.2721494107D 01
           XB(  5) = X(  7) =      0.5264347680D 00
           XB(  6) = X(  1) =      0.2270022283D 00
           XB(  7) = X(  9) =      0.6567301725D 00
           XB(  8) = X( 51) =      0.8402533881D 00
           XB(  9) = X(  8) =      0.1174241027D 00
           XB( 10) = X( 15) =      0.1597466119D 00
      THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =     -0.84025D 00
      THE CURRENT BASIS INVERSE IS AS FOLLOWS:
        THE  1-TH ROW:     0.10000000D 01     -0.00000000D 00     -0.10000000D 01      0.10000000D 01     -0.10000000D 01
-0.10000000D 01     -0.10000000D 01     -0.00000000D 00      0.30000000D 01     -0.00000000D 00
        THE  2-TH ROW:     0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00      0.10000000D 01     -0.10000000D 01     -0.00000000D 00
        THE  3-TH ROW:     0.00000000D 00     -0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
        THE  4-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00      0.10000000D 01     -0.00000000D 00
        THE  5-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00      0.10000000D 01
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
        THE  6-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
        THE  7-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
        THE  8-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.10000000D 01      0.10000000D 01     -0.00000000D 00
        THE  9-TH ROW:     0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00
 0.10000000D 01      0.00000000D 00      0.00000000D 00     -0.10000000D 01      0.00000000D 00
        THE 10-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00      0.10000000D 01
                    PHASE 1 :  ITERATION   4
      THE REDUCED COSTS
           THE   1-TH REDUCED COST =    -0.0000000000D 00
           THE   2-TH REDUCED COST =     0.1000000000D 01
           THE   3-TH REDUCED COST =    -0.0000000000D 00
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        THE   4-TH REDUCED COST =    -0.1000000000D 01
        THE   5-TH REDUCED COST =    -0.0000000000D 00
        THE   6-TH REDUCED COST =    -0.0000000000D 00
        THE   7-TH REDUCED COST =    -0.0000000000D 00
        THE   8-TH REDUCED COST =    -0.0000000000D 00
        THE   9-TH REDUCED COST =    -0.0000000000D 00
        THE  10-TH REDUCED COST =    -0.0000000000D 00
        THE  11-TH REDUCED COST =    -0.0000000000D 00
        THE  12-TH REDUCED COST =    -0.0000000000D 00
        THE  13-TH REDUCED COST =    -0.0000000000D 00
        THE  14-TH REDUCED COST =    -0.0000000000D 00
        THE  15-TH REDUCED COST =    -0.1000000000D 01
        THE  16-TH REDUCED COST =    -0.0000000000D 00
    X(  2) IS THE NON-BASIC VARIABLE WITH THE SMALLEST INDEX
    WHICH HAS THE LARGEST POSITIVE REDUCED COST.
    THEREFORE X(  2) WILL ENTER THE BASIS.
    THE COMPONENTS OF Y(-,  2)
        Y(  3,  2) =    -0.1120000000D 01
        Y(  4,  2) =     0.1950000000D 01
        Y(  5,  2) =    -0.3020000000D 01
        Y(  6,  2) =     0.0000000000D 00
        Y(  7,  2) =    -0.6590000000D 01
        Y(  8,  2) =     0.1000000000D 01
        Y(  9,  2) =    -0.5430000000D 01
        Y( 10,  2) =    -0.1000000000D 01
    THE VECTOR TO LEAVE THE BASIS IS NOW DETERMINED.
    THE XB(I)/Y(I,  2) RATIOS WITH Y(I,  2) > 0. ARE:
           0.13956D 01= XB(  4)/Y(  4,  2)
           0.84025D 00= XB(  8)/Y(  8,  2)
    THE MINIMUM RATIO IS XB(  8)/Y(  8,  2) =      0.8402533881D 00.
    THEREFORE THE   8-TH VARIABLE LEAVES THE BASIS.
    THE BASIC VARIABLES ARE NOW
          0,50, 5,11, 7, 1, 9, 2, 8,15,
    THE VALUES OF THE BASIC VARIABLES ARE NOW
         XB(  1) = X(  0) =     -0.1851800000D 02
         XB(  2) = X( 50) =     -0.0000000000D 00
         XB(  3) = X(  5) =      0.3497000000D 01
         XB(  4) = X( 11) =      0.1083000000D 01
         XB(  5) = X(  7) =      0.3064000000D 01
         XB(  6) = X(  1) =      0.2270022283D 00
         XB(  7) = X(  9) =      0.6194000000D 01
         XB(  8) = X(  2) =      0.8402533881D 00
         XB(  9) = X(  8) =      0.4680000000D 01
         XB( 10) = X( 15) =      0.1000000000D 01
    THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.00000D 00
    THE CURRENT BASIS INVERSE IS AS FOLLOWS:
       THE  1-TH ROW:     0.10000000D 01     -0.00000000D 00     -0.10000000D 01      0.10000000D 01     -0.10000000D 01
-0.10000000D 01     -0.10000000D 01      0.14210000D 02     -0.11210000D 02     -0.00000000D 00
       THE  2-TH ROW:     0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
       THE  3-TH ROW:     0.00000000D 00     -0.00000000D 00      0.10000000D 01     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.11200000D 01      0.12000000D 00     -0.00000000D 00
       THE  4-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00      0.19500000D 01     -0.95000000D 00     -0.00000000D 00
       THE  5-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00      0.10000000D 01
-0.00000000D 00     -0.00000000D 00     -0.30200000D 01      0.20200000D 01     -0.00000000D 00
       THE  6-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
       THE  7-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00      0.10000000D 01     -0.65900000D 01      0.55900000D 01     -0.00000000D 00
       THE  8-TH ROW:     0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00      0.00000000D 00
 0.00000000D 00      0.00000000D 00     -0.10000000D 01      0.10000000D 01      0.00000000D 00
       THE  9-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
 0.10000000D 01     -0.00000000D 00     -0.54300000D 01      0.44300000D 01     -0.00000000D 00
       THE 10-TH ROW:     0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.10000000D 01      0.10000000D 01      0.10000000D 01
                      PHASE 1 :  ITERATION   5
    THE REDUCED COSTS
        THE   1-TH REDUCED COST =    -0.0000000000D 00
        THE   2-TH REDUCED COST =    -0.0000000000D 00
        THE   3-TH REDUCED COST =    -0.0000000000D 00
        THE   4-TH REDUCED COST =    -0.0000000000D 00
        THE   5-TH REDUCED COST =    -0.0000000000D 00
        THE   6-TH REDUCED COST =    -0.0000000000D 00
        THE   7-TH REDUCED COST =    -0.0000000000D 00
        THE   8-TH REDUCED COST =    -0.0000000000D 00
        THE   9-TH REDUCED COST =    -0.0000000000D 00
        THE  10-TH REDUCED COST =    -0.0000000000D 00
        THE  11-TH REDUCED COST =    -0.0000000000D 00
        THE  12-TH REDUCED COST =    -0.0000000000D 00
        THE  13-TH REDUCED COST =    -0.0000000000D 00
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          THE  14-TH REDUCED COST =    -0.0000000000D 00
          THE  15-TH REDUCED COST =    -0.0000000000D 00
          THE  16-TH REDUCED COST =    -0.0000000000D 00
     THE VALUE OF THE SUM OF THE ARTIFICIAL VARIABLES AT THE END OF PHASE 1
     IS ZERO.  (ACTUALLY IT IS     0.0000000000D 00)

         PHASE 2 NOW BEGINS.


     THE INITIAL FEASIBLE SOLUTION IS AS FOLLOWS:
          X(  1) =      0.2270022283D 00
          X(  2) =      0.8402533881D 00
          X(  3) =      0.0000000000D 00
          X(  4) =      0.0000000000D 00
          X(  5) =      0.3497000000D 01
          X(  6) =      0.0000000000D 00
          X(  7) =      0.3064000000D 01
          X(  8) =      0.4680000000D 01
          X(  9) =      0.6194000000D 01
          X( 10) =      0.0000000000D 00
          X( 11) =      0.1083000000D 01
          X( 12) =      0.0000000000D 00
          X( 13) =      0.0000000000D 00
          X( 14) =      0.0000000000D 00
          X( 15) =      0.1000000000D 01
          X( 16) =      0.0000000000D 00
     THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.18518D 02
                    PHASE 2 :  ITERATION   1
     THE REDUCED COSTS
          THE   1-TH REDUCED COST =    -0.2220446049D-15
          THE   2-TH REDUCED COST =    -0.0000000000D 00
          THE   3-TH REDUCED COST =     0.2220446049D-15
          THE   4-TH REDUCED COST =    -0.0000000000D 00
          THE   5-TH REDUCED COST =    -0.0000000000D 00
          THE   6-TH REDUCED COST =    -0.2000000000D 01
          THE   7-TH REDUCED COST =    -0.0000000000D 00
          THE   8-TH REDUCED COST =    -0.0000000000D 00
          THE   9-TH REDUCED COST =    -0.0000000000D 00
          THE  10-TH REDUCED COST =    -0.2000000000D 01
          THE  11-TH REDUCED COST =    -0.0000000000D 00
          THE  12-TH REDUCED COST =    -0.2000000000D 01
          THE  13-TH REDUCED COST =    -0.2000000000D 01
          THE  14-TH REDUCED COST =    -0.2000000000D 01
          THE  15-TH REDUCED COST =    -0.1121000000D 02
          THE  16-TH REDUCED COST =    -0.0000000000D 00
     ALL OF THE REDUCED COSTS ARE NON-POSITIVE.
     THEREFORE THE CURRENT BASIC FEASIBLE SOLUTION IS AN OPTIMAL SOLUTION.
     THE OPTIMAL SOLUTION IS AS FOLLOWS:
          X(  1) =      0.2270022283D 00
          X(  2) =      0.8402533881D 00
          X(  3) =      0.0000000000D 00
          X(  4) =      0.0000000000D 00
          X(  5) =      0.3497000000D 01
          X(  6) =      0.0000000000D 00
          X(  7) =      0.3064000000D 01
          X(  8) =      0.4680000000D 01
          X(  9) =      0.6194000000D 01
          X( 10) =      0.0000000000D 00
          X( 11) =      0.1083000000D 01
          X( 12) =      0.0000000000D 00
          X( 13) =      0.0000000000D 00
          X( 14) =      0.0000000000D 00
          X( 15) =      0.1000000000D 01
          X( 16) =      0.0000000000D 00
     THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.18518D 02

  THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0, OF THE REGRESSION PARAMETER VECTOR, BETA
        LEAST SQUARES ESTIMATE OF BETA(  1) =        3.227002
        LEAST SQUARES ESTIMATE OF BETA(  2) =       -1.159747
  MR.A'S ANSWER :            THE ESTIMATE OF THE REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE ABSOLUTE RESIDUALS
        L1 ESTIMATE OF BETA(  1) =        3.000000
        L1 ESTIMATE OF BETA(  2) =       -2.000000
  THE RESIDUALS :
        THE   1-TH RESIDUAL =       3.497000
        THE   2-TH RESIDUAL =      -1.083000
        THE   3-TH RESIDUAL =       3.064000
        THE   4-TH RESIDUAL =       4.680000
        THE   5-TH RESIDUAL =       6.194000
  THE SUM OF THE ABSOLUTE RESIDUALS =       18.518000
  THE MAXIMUM ABSOLUTE RESIDUAL =       6.194000
THE MONTE CARLO STUDY TO ESTIMATE THE COVARIANCE OF THE L1 ESTIMATE OF BETA NOW BEGINS
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RESULTS FOR MONTE CARLO SAMPLE NUMBER   1:
 THE PROBLEM P2 HAS BEEN SELECTED
 THE INVERSE OF XTX
      0.810298     -0.168497
     -0.168497      0.046521
 THE VALUE OF BETAO TO COMPUTE RHS
     -0.594250      0.136933
THE RIGHT HAND SIDES FOR THIS SAMPLE
   BRHS1(  1) =       0.000000
   BRHS1(  2) =       0.000000
   BRHS1(  3) =       0.067696
   BRHS1(  4) =      -0.244174
   BRHS1(  5) =       0.226595
   BRHS1(  6) =      -0.039891
   BRHS1(  7) =      -0.010226
   BRHS1(  8) =       0.236092
   BRHS1(  9) =       0.582285
   BRHS1( 10) =      -0.184188
THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE
   BETA(  1) =      -0.594250
   BETA(  2) =       0.136933
THE EPSILONS GENERATED FOR THIS SAMPLE
   EPSILON(  1) =      -0.373189
   EPSILON(  2) =      -0.571406
   EPSILON(  3) =       0.045882
   EPSILON(  4) =       0.109403
   EPSILON(  5) =       0.297911
 THE INITIAL VALUES OF THE BASIC VARIABLES
      XB(  0) =    -0.36609D 01
      XB(  1) =     0.00000D 00
      XB(  2) =    -0.12685D 00
      XB(  3) =     0.15138D 00
      XB(  4) =     0.68981D 00
      XB(  5) =    -0.58228D 00
      XB(  6) =     0.16889D 01
      XB(  7) =     0.34619D 00
      XB(  8) =     0.12577D 01
      XB(  9) =     0.16200D 00
 THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =    -0.36609D 01
 THE REDUCED COSTS
      THE    1-TH REDUCED COST =    -0.00000D 00
      THE    2-TH REDUCED COST =    -0.00000D 00
      THE    3-TH REDUCED COST =     0.22204D-15
      THE    4-TH REDUCED COST =    -0.00000D 00
      THE    5-TH REDUCED COST =    -0.00000D 00
      THE    6-TH REDUCED COST =    -0.20000D 01
      THE    7-TH REDUCED COST =    -0.00000D 00
      THE    8-TH REDUCED COST =    -0.00000D 00
      THE    9-TH REDUCED COST =    -0.00000D 00
      THE   10-TH REDUCED COST =    -0.20000D 01
      THE   11-TH REDUCED COST =    -0.00000D 00
      THE   12-TH REDUCED COST =    -0.20000D 01
      THE   13-TH REDUCED COST =    -0.20000D 01
      THE   14-TH REDUCED COST =    -0.20000D 01
      THE   15-TH REDUCED COST =    -0.00000D 00
      THE   16-TH REDUCED COST =    -0.00000D 00
 TENTATIVELY THE    6-TH BASIC VARIABLE IS LEAVING THE BASIS
 THE YRJ'S
 (  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
 IT CAN BE ZERO OTHERWISE TOO.)
      YR(  1) =     0.00000D 00
      YR(  2) =     0.00000D 00
      YR(  3) =    -0.10000D 01
      YR(  4) =     0.00000D 00
      YR(  5) =     0.00000D 00
      YR(  6) =     0.00000D 00
      YR(  7) =     0.00000D 00
      YR(  8) =     0.00000D 00
      YR(  9) =     0.00000D 00
      YR( 10) =     0.00000D 00
      YR( 11) =     0.00000D 00
      YR( 12) =     0.00000D 00
      YR( 13) =     0.00000D 00
      YR( 14) =     0.00000D 00
      YR( 15) =     0.00000D 00
      YR( 16) =     0.00000D 00
 THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
      THE RATIO OF THE    3-TH NET PRICE TO YR(  3) =      0.22204D-15
 THE    6-TH VARIABLE IS LEAVING THE BASIS.
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THE   3-TH VARIABLE IS ENTERING THE BASIS.
THE CURRENT BASIS INVERSE IS AS FOLLOWS:
  THE  1-TH ROW:     0.10000000D 01    -0.00000000D 00    -0.10000000D 01     0.10000000D 01    -0.10000000D 01
-0.10000000D 01    -0.10000000D 01     0.14210000D 02    -0.11210000D 02    -0.00000000D 00
  THE  2-TH ROW:     0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
  THE  3-TH ROW:     0.00000000D 00    -0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.11200000D 01     0.12000000D 00    -0.00000000D 00
  THE  4-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.10000000D 01    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00     0.19500000D 01    -0.95000000D 00    -0.00000000D 00
  THE  5-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01
-0.00000000D 00    -0.00000000D 00    -0.30200000D 01     0.20200000D 01    -0.00000000D 00
  THE  6-TH ROW:     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00     0.00000000D 00
 0.00000000D 00     0.00000000D 00     0.00000000D 00     0.10000000D 01     0.00000000D 00
  THE  7-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00     0.10000000D 01    -0.65900000D 01     0.55900000D 01    -0.00000000D 00
  THE  8-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.10000000D 01     0.10000000D 01    -0.00000000D 00
  THE  9-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
 0.10000000D 01    -0.00000000D 00    -0.54300000D 01     0.44300000D 01    -0.00000000D 00
  THE 10-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.10000000D 01     0.10000000D 01     0.10000000D 01
THE BASIC VARIABLES ARE NOW
       0,  50,   5,  11,   7,   3,   9,   2,   8,  15,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =    -0.00000D 00
     XB(  2) =    -0.12685D 00
     XB(  3) =     0.15138D 00
     XB(  4) =     0.68981D 00
     XB(  5) =     0.58228D 00
     XB(  6) =     0.16889D 01
     XB(  7) =     0.34619D 00
     XB(  8) =     0.12577D 01
     XB(  9) =     0.16200D 00
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.36609D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.00000D 00
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =    -0.00000D 00
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.20000D 01
     THE   7-TH REDUCED COST =    -0.00000D 00
     THE   8-TH REDUCED COST =    -0.00000D 00
     THE   9-TH REDUCED COST =    -0.00000D 00
     THE  10-TH REDUCED COST =    -0.20000D 01
     THE  11-TH REDUCED COST =    -0.00000D 00
     THE  12-TH REDUCED COST =    -0.20000D 01
     THE  13-TH REDUCED COST =    -0.20000D 01
     THE  14-TH REDUCED COST =    -0.20000D 01
     THE  15-TH REDUCED COST =    -0.00000D 00
     THE  16-TH REDUCED COST =    -0.00000D 00
TENTATIVELY THE   3-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     0.00000D 00
     YR(  2) =     0.00000D 00
     YR(  3) =     0.00000D 00
     YR(  4) =     0.00000D 00
     YR(  5) =     0.00000D 00
     YR(  6) =     0.00000D 00
     YR(  7) =     0.00000D 00
     YR(  8) =     0.00000D 00
     YR(  9) =     0.00000D 00
     YR( 10) =    -0.10000D 01
     YR( 11) =     0.00000D 00
     YR( 12) =     0.00000D 00
     YR( 13) =     0.00000D 00
     YR( 14) =     0.00000D 00
     YR( 15) =     0.00000D 00
     YR( 16) =     0.00000D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE  10-TH NET PRICE TO YR( 10) =    -0.20000D 01
THE   3-TH VARIABLE IS LEAVING THE BASIS.
THE  10-TH VARIABLE IS ENTERING THE BASIS.
THE CURRENT BASIS INVERSE IS AS FOLLOWS:
  THE  1-TH ROW:     0.10000000D 01    -0.00000000D 00     0.10000000D 01     0.10000000D 01    -0.10000000D 01
-0.10000000D 01    -0.10000000D 01     0.11970000D 02    -0.10970000D 02    -0.00000000D 00
  THE  2-TH ROW:     0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
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        THE  3-TH ROW:     0.00000000D 00      0.00000000D 00      -0.10000000D 01      0.00000000D 00      0.00000000D 00
 0.00000000D 00      0.00000000D 00      0.11200000D 01      -0.12000000D 00      0.00000000D 00
        THE  4-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.10000000D 01     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00      0.19500000D 01      -0.95000000D 00     -0.00000000D 00
        THE  5-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.00000000D 00      0.10000000D 01
-0.00000000D 00     -0.00000000D 00     -0.30200000D 01       0.20200000D 01     -0.00000000D 00
        THE  6-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.00000000D 00       0.10000000D 01     -0.00000000D 00
        THE  7-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00      0.10000000D 01     -0.65900000D 01       0.55900000D 01     -0.00000000D 00
        THE  8-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.10000000D 01       0.10000000D 01     -0.00000000D 00
        THE  9-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
 0.10000000D 01     -0.00000000D 00     -0.54300000D 01       0.44300000D 01     -0.00000000D 00
        THE 10-TH ROW:     0.00000000D 00     -0.00000000D 00      -0.00000000D 00     -0.00000000D 00     -0.00000000D 00
-0.00000000D 00     -0.00000000D 00     -0.10000000D 01       0.10000000D 01      0.10000000D 01
      THE BASIC VARIABLES ARE NOW
          0,  50,  10,  11,   7,   3,   9,   2,   8,  15,
      THE VALUES OF THE BASIC VARIABLES ARE NOW
          XB(  1) =    -0.00000D 00
          XB(  2) =     0.12685D 00
          XB(  3) =     0.15138D 00
          XB(  4) =     0.68981D 00
          XB(  5) =     0.58228D 00
          XB(  6) =     0.16889D 01
          XB(  7) =     0.34619D 00
          XB(  8) =     0.12577D 01
          XB(  9) =     0.16200D 00
      THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.39146D 01
      THE REDUCED COSTS
          THE   1-TH REDUCED COST =    -0.00000D 00
          THE   2-TH REDUCED COST =    -0.00000D 00
          THE   3-TH REDUCED COST =    -0.00000D 00
          THE   4-TH REDUCED COST =    -0.00000D 00
          THE   5-TH REDUCED COST =    -0.20000D 01
          THE   6-TH REDUCED COST =    -0.20000D 01
          THE   7-TH REDUCED COST =    -0.00000D 00
          THE   8-TH REDUCED COST =    -0.00000D 00
          THE   9-TH REDUCED COST =    -0.00000D 00
          THE  10-TH REDUCED COST =    -0.00000D 00
          THE  11-TH REDUCED COST =    -0.00000D 00
          THE  12-TH REDUCED COST =    -0.20000D 01
          THE  13-TH REDUCED COST =    -0.20000D 01
          THE  14-TH REDUCED COST =    -0.20000D 01
          THE  15-TH REDUCED COST =    -0.00000D 00
          THE  16-TH REDUCED COST =    -0.00000D 00
   THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
   THE NONZERO VARIABLES ARE AS FOLLOWS:
          X(  0) =    -0.39146D 01
          X( 50) =    -0.00000D 00
          X( 10) =     0.12685D 00
          X( 11) =     0.15138D 00
          X(  7) =     0.68981D 00
          X(  3) =     0.58228D 00
          X(  9) =     0.16889D 01
          X(  2) =     0.34619D 00
          X(  8) =     0.12577D 01
          X( 15) =     0.16200D 00
   THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.39146D 01

     RESULTS FOR MONTE CARLO SAMPLE NUMBER   2:
      THE PROBLEM P1 HAS BEEN SELECTED
      THE INVERSE OF XTX
          0.810298    -0.168497
         -0.168497     0.046521
      THE VALUE OF BETA0 TO COMPUTE RHS
          0.217174     0.114161
     THE RIGHT HAND SIDES FOR THIS SAMPLE
        BRHS1(  1) =       0.000000
        BRHS1(  2) =       0.000000
        BRHS1(  3) =       0.534109
        BRHS1(  4) =       0.157815
        BRHS1(  5) =      -0.919343
        BRHS1(  6) =      -0.320504
        BRHS1(  7) =       0.547923
        BRHS1(  8) =       0.070676
        BRHS1(  9) =       0.294137
        BRHS1( 10) =      -0.117764
     THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE
        BETA(  1) =      0.217174
        BETA(  2) =      0.114161
```

(Etc.)

```
      BRHS1(  3) =       -1.062358
      BRHS1(  4) =        1.453735
      BRHS1(  5) =       -0.206058
      BRHS1(  6) =       -0.171215
      BRHS1(  7) =       -0.014105
      BRHS1(  8) =        0.358040
      BRHS1(  9) =        0.985521
      BRHS1( 10) =       -0.297676
   THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE
      BETA(  1) =       0.011806
      BETA(  2) =      -0.097417
   THE EPSILONS GENERATED FOR THIS SAMPLE
      EPSILON(  1) =      -1.159658
      EPSILON(  2) =       1.275578
      EPSILON(  3) =      -0.488451
      EPSILON(  4) =      -0.688384
      EPSILON(  5) =      -0.644277
    THE INITIAL VALUES OF THE BASIC VARIABLES
         XB(  0) =     -0.91263D 01
         XB(  1) =      0.00000D 00
         XB(  2) =      0.13451D 01
         XB(  3) =      0.16918D 01
         XB(  4) =      0.70341D 00
         XB(  5) =      0.98552D 00
         XB(  6) =      0.31355D 01
         XB(  7) =      0.62748D 00
         XB(  8) =      0.22505D 01
         XB(  9) =      0.32980D 00
    THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =    -0.91263D 01
    THE REDUCED COSTS
         THE   1-TH REDUCED COST =    -0.00000D 00
         THE   2-TH REDUCED COST =     0.28866D-14
         THE   3-TH REDUCED COST =    -0.28866D-14
         THE   4-TH REDUCED COST =    -0.00000D 00
         THE   5-TH REDUCED COST =    -0.20000D 01
         THE   6-TH REDUCED COST =    -0.00000D 00
         THE   7-TH REDUCED COST =    -0.00000D 00
         THE   8-TH REDUCED COST =    -0.00000D 00
         THE   9-TH REDUCED COST =    -0.00000D 00
         THE  10-TH REDUCED COST =    -0.00000D 00
         THE  11-TH REDUCED COST =    -0.20000D 01
         THE  12-TH REDUCED COST =    -0.20000D 01
         THE  13-TH REDUCED COST =    -0.20000D 01
         THE  14-TH REDUCED COST =    -0.20000D 01
         THE  15-TH REDUCED COST =    -0.00000D 00
         THE  16-TH REDUCED COST =    -0.00000D 00
THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
THE NONZERO VARIABLES ARE AS FOLLOWS:
         X(  0) =    -0.91263D 01
         X( 50) =     0.00000D 00
         X( 10) =     0.13451D 01
         X(  6) =     0.16918D 01
         X(  7) =     0.70341D 00
         X(  1) =     0.98552D 00
         X(  9) =     0.31355D 01
         X(  4) =     0.62748D 00
         X(  8) =     0.22505D 01
         X( 15) =     0.32980D 00
THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.91263D 01

   RESULTS FOR MONTE CARLO SAMPLE NUMBER  20:
    THE PROBLEM P2 HAS BEEN SELECTED
    THE INVERSE OF XTX
         0.810298     -0.168497
        -0.168497      0.046521
    THE VALUE OF BETA0 TO COMPUTE RHS
         0.173768     -0.207514
   THE RIGHT HAND SIDES FOR THIS SAMPLE
      BRHS1(  1) =        0.000000
      BRHS1(  2) =        0.000000
      BRHS1(  3) =        0.478624
      BRHS1(  4) =       -0.259669
      BRHS1(  5) =       -0.702085
      BRHS1(  6) =        0.942444
      BRHS1(  7) =       -0.459314
      BRHS1(  8) =        0.349361
      BRHS1(  9) =        0.955143
      BRHS1( 10) =       -0.267826
   THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE
      BETA(  1) =       0.173768
      BETA(  2) =      -0.207514
```

(RECONTINUED)

```
THE EPSILONS GENERATED FOR THIS SAMPLE
   EPSILON(  1) =       0.419977
   EPSILON(  2) =      -0.490553
   EPSILON(  3) =      -1.155009
   EPSILON(  4) =      -0.010589
   EPSILON(  5) =      -1.653063
 THE INITIAL VALUES OF THE BASIC VARIABLES
      XB(  0) =    -0.57911D 01
      XB(  1) =     0.00000D 00
      XB(  2) =    -0.20196D 00
      XB(  3) =    -0.33537D-01
      XB(  4) =     0.17223D 00
      XB(  5) =     0.95514D 00
      XB(  6) =     0.25776D 01
      XB(  7) =     0.60578D 00
      XB(  8) =     0.32767D 01
      XB(  9) =     0.33796D 00
 THE INITIAL VALUE OF THE OBJECTIVE FUNCTION =    -0.57911D 01
 THE REDUCED COSTS
      THE   1-TH REDUCED COST =    -0.28866D-14
      THE   2-TH REDUCED COST =    -0.00000D 00
      THE   3-TH REDUCED COST =    -0.00000D 00
      THE   4-TH REDUCED COST =     0.28866D-14
      THE   5-TH REDUCED COST =    -0.20000D 01
      THE   6-TH REDUCED COST =    -0.00000D 00
      THE   7-TH REDUCED COST =    -0.00000D 00
      THE   8-TH REDUCED COST =    -0.00000D 00
      THE   9-TH REDUCED COST =    -0.00000D 00
      THE  10-TH REDUCED COST =    -0.00000D 00
      THE  11-TH REDUCED COST =    -0.20000D 01
      THE  12-TH REDUCED COST =    -0.20000D 01
      THE  13-TH REDUCED COST =    -0.20000D 01
      THE  14-TH REDUCED COST =    -0.20000D 01
      THE  15-TH REDUCED COST =    -0.00000D 00
      THE  16-TH REDUCED COST =    -0.00000D 00
 TENTATIVELY THE   3-TH BASIC VARIABLE IS LEAVING THE BASIS
 THE YRJ'S
 (  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
 IT CAN BE ZERO OTHERWISE TOO.)
      YR(  1) =     0.00000D 00
      YR(  2) =     0.00000D 00
      YR(  3) =     0.00000D 00
      YR(  4) =     0.00000D 00
      YR(  5) =    -0.10000D 01
      YR(  6) =     0.00000D 00
      YR(  7) =     0.00000D 00
      YR(  8) =     0.00000D 00
      YR(  9) =     0.00000D 00
      YR( 10) =     0.00000D 00
      YR( 11) =     0.00000D 00
      YR( 12) =     0.00000D 00
      YR( 13) =     0.00000D 00
      YR( 14) =     0.00000D 00
      YR( 15) =     0.00000D 00
      YR( 16) =     0.00000D 00
 THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
      THE RATIO OF THE   5-TH NET PRICE TO YR(  5) =    -0.20000D 01
 THE   3-TH VARIABLE IS LEAVING THE BASIS.
 THE   5-TH VARIABLE IS ENTERING THE BASIS.
 THE CURRENT BASIS INVERSE IS AS FOLLOWS:
   THE  1-TH ROW:     0.10000000D 01    -0.00000000D 00    -0.10000000D 01    -0.10000000D 01    -0.10000000D 01
-0.10000000D 01    -0.10000000D 01     0.18110000D 02    -0.13110000D 02    -0.00000000D 00
   THE  2-TH ROW:     0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
   THE  3-TH ROW:     0.00000000D 00     0.00000000D 00     0.10000000D 01     0.00000000D 00     0.00000000D 00
 0.00000000D 00     0.00000000D 00    -0.11200000D 01     0.12000000D 00     0.00000000D 00
   THE  4-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.19500000D 01     0.95000000D 00    -0.00000000D 00
   THE  5-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01
-0.00000000D 00    -0.00000000D 00    -0.30200000D 01     0.20200000D 01    -0.00000000D 00
   THE  6-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01    -0.00000000D 00
   THE  7-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00     0.10000000D 01    -0.65900000D 01     0.55900000D 01    -0.00000000D 00
   THE  8-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.10000000D 01     0.10000000D 01    -0.00000000D 00
   THE  9-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
 0.10000000D 01    -0.00000000D 00    -0.54300000D 01     0.44300000D 01    -0.00000000D 00
   THE 10-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.10000000D 01     0.10000000D 01     0.10000000D 01
 THE BASIC VARIABLES ARE NOW
```

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        0,  50,   5,   6,   7,   3,   9,   2,   8,  15,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =    -0.00000D 00
     XB(  2) =     0.20196D 00
     XB(  3) =    -0.33537D-01
     XB(  4) =     0.17223D 00
     XB(  5) =     0.95514D 00
     XB(  6) =     0.25776D 01
     XB(  7) =     0.60578D 00
     XB(  8) =     0.32767D 01
     XB(  9) =     0.33796D 00
THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.61950D 01
THE REDUCED COSTS
     THE   1-TH REDUCED COST =    -0.28866D-14
     THE   2-TH REDUCED COST =    -0.00000D 00
     THE   3-TH REDUCED COST =    -0.00000D 00
     THE   4-TH REDUCED COST =    -0.00000D 00
     THE   5-TH REDUCED COST =    -0.00000D 00
     THE   6-TH REDUCED COST =    -0.00000D 00
     THE   7-TH REDUCED COST =    -0.00000D 00
     THE   8-TH REDUCED COST =    -0.00000D 00
     THE   9-TH REDUCED COST =    -0.00000D 00
     THE  10-TH REDUCED COST =    -0.20000D 01
     THE  11-TH REDUCED COST =    -0.20000D 01
     THE  12-TH REDUCED COST =    -0.20000D 01
     THE  13-TH REDUCED COST =    -0.20000D 01
     THE  14-TH REDUCED COST =    -0.20000D 01
     THE  15-TH REDUCED COST =    -0.00000D 00
     THE  16-TH REDUCED COST =    -0.00000D 00
TENTATIVELY THE   4-TH BASIC VARIABLE IS LEAVING THE BASIS
THE YRJ'S
(  YR(J) = 0 IF THE J-TH VARIABLE IS IN THE BASIS.
IT CAN BE ZERO OTHERWISE TOO.)
     YR(  1) =     0.00000D 00
     YR(  2) =     0.00000D 00
     YR(  3) =     0.00000D 00
     YR(  4) =     0.00000D 00
     YR(  5) =     0.00000D 00
     YR(  6) =     0.00000D 00
     YR(  7) =     0.00000D 00
     YR(  8) =     0.00000D 00
     YR(  9) =     0.00000D 00
     YR( 10) =     0.00000D 00
     YR( 11) =    -0.10000D 01
     YR( 12) =     0.00000D 00
     YR( 13) =     0.00000D 00
     YR( 14) =     0.00000D 00
     YR( 15) =     0.00000D 00
     YR( 16) =     0.00000D 00
THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ'S
     THE RATIO OF THE  11-TH NET PRICE TO YR( 11) =    -0.20000D 01
THE   4-TH VARIABLE IS LEAVING THE BASIS.
THE  11-TH VARIABLE IS ENTERING THE BASIS.
THE CURRENT BASIS INVERSE IS AS FOLLOWS:
  THE  1-TH ROW:     0.10000000D 01    -0.00000000D 00    -0.10000000D 01     0.10000000D 01    -0.10000000D 01
-0.10000000D 01    -0.10000000D 01     0.14210000D 02    -0.11210000D 02    -0.00000000D 00
  THE  2-TH ROW:     0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
  THE  3-TH ROW:     0.00000000D 00    -0.00000000D 00     0.10000000D 01    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.11200000D 01     0.12000000D 00    -0.00000000D 00
  THE  4-TH ROW:     0.00000000D 00     0.00000000D 00     0.00000000D 00    -0.10000000D 01     0.00000000D 00
 0.00000000D 00     0.00000000D 00     0.19500000D 01    -0.95000000D 00     0.00000000D 00
  THE  5-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01
-0.00000000D 00    -0.00000000D 00    -0.30200000D 01     0.20200000D 01    -0.00000000D 00
  THE  6-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.00000000D 00     0.10000000D 01    -0.00000000D 00
  THE  7-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00     0.10000000D 01    -0.65900000D 01     0.55900000D 01    -0.00000000D 00
  THE  8-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.10000000D 01     0.10000000D 01    -0.00000000D 00
  THE  9-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
 0.10000000D 01    -0.00000000D 00    -0.54300000D 01     0.44300000D 01    -0.00000000D 00
  THE 10-TH ROW:     0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00    -0.00000000D 00
-0.00000000D 00    -0.00000000D 00    -0.10000000D 01     0.10000000D 01     0.10000000D 01
THE BASIC VARIABLES ARE NOW
        0,  50,   5,  11,   7,   3,   9,   2,   8,  15,
THE VALUES OF THE BASIC VARIABLES ARE NOW
     XB(  1) =    -0.00000D 00
     XB(  2) =     0.20196D 00
     XB(  3) =     0.33537D-01
     XB(  4) =     0.17223D 00
```

```
          XB(  5) =      0.95514D 00
          XB(  6) =      0.25776D 01
          XB(  7) =      0.60578D 00
          XB(  8) =      0.32767D 01
          XB(  9) =      0.33796D 00
      THE CURRENT VALUE OF THE OBJECTIVE FUNCTION =    -0.62621D 01
      THE REDUCED COSTS
          THE   1-TH REDUCED COST =    -0.28866D-14
          THE   2-TH REDUCED COST =    -0.00000D 00
          THE   3-TH REDUCED COST =    -0.00000D 00
          THE   4-TH REDUCED COST =     0.28866D-14
          THE   5-TH REDUCED COST =    -0.00000D 00
          THE   6-TH REDUCED COST =    -0.20000D 01
          THE   7-TH REDUCED COST =    -0.00000D 00
          THE   8-T  .EDUCED COST =    -0.00000D 00
          THE   9-iH REDUCED COST =    -0.00000D 00
          THE  10-TH REDUCED COST =    -0.20000D 01
          THE  11-TH REDUCED COST      -0.00000D 00
          THE  12-TH REDUCED COST =    -0.20000D 01
          THE  13-TH REDUCED COST =    -0.20000D 01
          THE  14-TH REDUCED COST =    -0.20000D 01
          THE  15-TH REDUCED COST =    -0.00000D 00
          THE  16-TH REDUCED COST =    -0.00000D 00
  THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OPTIMAL.
  THE NONZERO VARIABLES ARE AS FOLLOWS:
          X(  0) =    -0.62621D 01
          X( 50) =    -0.00000D 00
          X(  5) =     0.20196D 00
          X( 11) =     0.33537D-01
          X(  7) =     0.17223D 00
          X(  3) =     0.95514D 00
          X(  9) =     0.25776D 01
          X(  2) =     0.60578D 00
          X(  8) =     0.32767D 01
          X( 15) =     0.33796D 00
  THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS    -0.62621D 01

  AUXILIARY RESULTS OF THE MINI MONTE CARLO STUDY

    VALUES OF DELTA BETA STAR
        SAMPLE NUMBER =  1       -0.01197   -0.20926
        SAMPLE NUMBER =  2        0.51131   -0.10930
        SAMPLE NUMBER =  3        1.43557   -0.71028
        SAMPLE NUMBER =  4        0.61193   -0.44646
        SAMPLE NUMBER =  5        0.61553   -0.51802
        SAMPLE NUMBER =  6       -0.28073   -0.21389
        SAMPLE NUMBER =  7       -0.18772   -0.34817
        SAMPLE NUMBER =  8       -0.36551   -0.05786
        SAMPLE NUMBER =  9        0.95646   -0.28508
        SAMPLE NUMBER = 10        0.03818   -0.28217
        SAMPLE NUMBER = 11        1.59676   -0.62135
        SAMPLE NUMBER = 12        0.62508   -0.34275
        SAMPLE NUMBER = 13       -0.10425   -0.41664
        SAMPLE NUMBER = 14        0.67201   -0.34557
        SAMPLE NUMBER = 15        1.18308   -0.61111
        SAMPLE NUMBER = 16        1.26476   -0.56381
        SAMPLE NUMBER = 17        0.66750   -0.66472
        SAMPLE NUMBER = 18        1.85856   -0.79734
        SAMPLE NUMBER = 19        0.99733   -0.72490
        SAMPLE NUMBER = 20        1.12891   -0.81330
    ESTIMATED COVARIANCE OF DELTA BETA STAR
             0.448991     -0.122225
            -0.122225      0.054857
    SUM OF THE OPTIMAL OBJECTIVE FUNCTIONS OVER ALL SAMPLES =    131.427570
  MAIN RESULTS OF THE MINI MONTE CARLO STUDY

  ESTIMATED VALUE OF SIGMA =       2.817978
  AUXILIARY RESULT : THE ESTIMATE OF SIGMA 5 =       7.736290
  ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER VECTOR, BETA
            3.565439     -0.970588
           -0.970588      0.435623
```

APPENDIX C. PROGRAM LISTING

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C
C     MR.A:     MINIMIZES THE ABSOLUTE RESIDUALS.
C
C     THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING THE
C     SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION, IN THE MODEL:
C               Y = X*BETA + EPSILON
C     WHERE,
C           Y = A VECTOR OF NOBS OBSERVATIONS,
C           X = AN NOBS BY IP MATRIX OF CONSTANTS,
C           BETA = AN IP LENGTH VECTOR OF UNKNOWN PARAMETERS
C                  SUBJECT TO RESTRICTIONS OF THE FORMS
C                      A*BETA = B OR
C                      A*BETA < OR = B OR
C                      A*BETA > OR = B.
C
C     THE L1 ESTIMATION PROBLEM IS SOLVED USING THE LINEAR PROGRAM
C     RE ISED SIMPLEX ALGORITHM : STANDARD FORM 2
C
C     THIS ALGORITHM ALWAYS BEGINS WITH AN ALL ARTIFICIAL BASIS
C     THE PRODUCT FORM OF THE BASIS INVERSE IS NOT USED
C
C     THE LINEAR PROGRAMMING PROBLEM IS ASSUMED TO BE IN THE FORM
C          MAX CX
C       SUBJECT TO
C          AX = BRHS
C          X GREATER THAN OR EQUAL TO 0
C     WHERE
C          BRHS IS A COLUMN OF NON-NEGATIVE CONSTANTS
C          A IS AN M-BY-N MATRIX OF CONSTANTS
C
C     THE AUGMENTED VERSIONS OF A,BRHS,B,XB,AND Y ARE REFERRED TO AS
C     A2,BRHS2,B2,XB2,AND Y2 RESPECTIVELY.
C
C
C      THE VARIANCE OF THE L1 ESTIMATOR IS ESTIMATED USING A MINI-
C      MONTE CARLO APPROACH INCORPORATING THE DUAL SIMPLEX
C      ALGORITHM.
C
C
C
C     THE FOLLOWING PROCEDURE WAS DEVELOPED BY :
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C
C     THE SUPPORT OF THE OFFICE OF NAVAL RESEARCH IS GRATEFULLY
C     ACKNOWLEDGED.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/VALUE/A(20,10),X(20,10),B(20),MP2,N,NC,NOBS,IP
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6
      COMMON/NFEED/NSEED
      DIMENSION C(72),BRHS2(42),A2(42,72),BRHS1(42),DINDX(42)
      DIMENSION XB2(42),Y2(42),REDCOS(72),B2INV(42,42)
      DIMENSION Y(20),ESTAR(20),RES1(20),RES2(20),RES(20),TITLE(20)
      DIMENSION BETA0(10),YMXB(20),BMAB(20),ITYPE(20),IRHS(20)
      DIMENSION DB1(10),DB2(10),DELB(10,100),DBETA0(10),DBX(135)
      DIMENSION INBASE(42),ISTAT(135),XSOL(135),XB1(42)
      DIMENSION BHAT(10),SUM(10),VARDBS(10,10),VARB(10,10),WT(40)
      INTEGER PHASE
C
C     THE DIMENSIONED ARRAYS HAVE THE FOLLOWING DIMENSIONS:
C          C(N),BRHS2(M+2),A2(M+2,N),INBASE(M+2),BRHS1(M+2),
C          B2INV(M+2,M+2),XB2(M+2),Y2(M+2),REDCOS(N),DINDX(M+2)
C          A(NC,IP),X(NOBS,IP),B(NC),Y(NOBS),ESTAR(NOBS),RES(NOBS)
C          BETA0(IP),YMXB(NOBS),BMAB(NC),ITYPE(NC),IRHS(NC)
C          DB1(IP),DB2(IP),DELB(IP,ISAM),DBETA(IP),DBX(N)
C          INBASE(MP2),ISTAT(N),XB2(MP2),XSOL(N),XB1(MP2)
C          BHAT(IP),SUM(IP),VARDBS(IP,IP),VARB(IP,IP),WT(2*NOBS)
C
C     THE FOLLOWING 'TOLERANCES' ARE USED IN THE ALGORITHM.
C     THEY MUST BE NON-NEGATIVE AND WOULD BE ZERO EXCEPT FOR THE
C     NUMERICAL INACCURACY OF THE COMPUTER.
C          TOLR1 : IF THE MAX REDUCED COST IS LESS THAN OR EQUAL TO
C                  TOLR1 THEN ALL REDUCED COSTS ARE CONSIDERED TO BE
C                  NON-POSITIVE.
C          TOLR2 : ANY COMPONENT Y(I,J) LESS THAN OR EQUAL TO TOLR2
C                  IS CONSIDERED NON-POSITIVE.
C          TOLR3 : IF AT THE END OF PHASE 1 THE SUM OF THE ARTIFICIAL
C                  VARIABLES IS GREATER THAN OR EQUAL TO TOLR3, THEN
C                  THE SUM IS CONSIDERED NON-ZERO AND PHASE 2 IS NOT
C                  BEGUN.  OTHERWISE, THE SUM OF THE ARTIFICIAL
C                  VARIABLES IS CONSIDERED TO HAVE BEEN DRIVEN TO
C                  ZERO AND PHASE 2 IS BEGUN.
      TOLR1=1.0D-07
      TOLR2=1.0D-07
      TOLR3=1.0D-05
C
C     THE INPUT  - CARD NUMBER ONE OF INPUT CARDS:
C     IP = THE LENGTH OF THE VECTOR OF UNKNOWN PARAMETERS.
C     NOBS = THE NUMBER OF OBSERVATIONS.
C     NC = THE NUMBER OF CONSTRAINTS PLACED UPON BETA.
C     NSEED = A TEN DIGIT RANDOM NUMBER LESS THAN 2147483647.
C     ISAM = THE NUMBER OF SAMPLES TO BE GENERATED FOR THE MINI
C            MONTE CARLO STUDY.
C
C     IWRIT0 = 1 IF THE INPUTTED PROBLEM IS TO BE DESCRIBED;
C              0 IF OTHERWISE
C
C     IWRIT1 = 1 IF THE MAIN RESULTS ARE TO BE DESCRIBED;
C              0 OTHERWISE
C
C     IWRIT2 = 1 IF THE INTERMEDIATE RESULTS ARE TO BE DESCRIBED;
C              0 OTHERWISE
C
C     IWRIT3 = 1 IF THE BASIS INVERSE IS TO BE DESCRIBED
C              0 OTHERWISE
C
C     IWRIT4 = 1 IF THE MAIN RESULTS OF THE MINI MONTE CARLO STUDY
C              ARE TO BE PRINTED.
C            = 0 OTHERWISE.
C
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C       IWRIT5 = 1 IF THE INTERMEDIATE RESULTS OF THE MINI MONTE
C                  CARLO STUDY ARE TO BE PRINTED.
C              = 0 OTHERWISE.
C
C       IWRIT6 = 1 IF THE AUXILIARY RESULTS OF THE VARIANCE
C                  CALCULATION, AND THE RESIDUALS ARE TO BE PRINTED.
C              = 0 OTHERWISE.
C
C       IOPTN = 1 IF THE RANDOM ERRORS GENERATED FOR THE MINI MONTE
C                 CARLO STUDY ARE TO BE NORMALLY DISTRIBUTED.
C             = 2 IF THE RANDOM ERRORS GENERATED FOR THE MINI MONTE
C                 CARLO STUDY ARE TO BE DOUBLE EXPONENTIAL.
C             = 3 IF THE RANDOM ERRORS GENERATED FOR THE MINI MONTE
C                 CARLO STUDY ARE TO BE UNIFORM.
C
C       IWT = 1 IF THE RESIDUALS ARE TO BE ASSIGNED WEIGHT COEFFICIENTS
C           = 0 IF RESIDUALS ARE NOT ASSIGNED WEIGHTS.
C
C       THE FINAL RESULTS ARE ALWAYS PRINTED
C
      READ(5,4810)IP,NOBS,NC,NSEED,ISAM,IWRIT0,IWRIT1,IWRIT2,IWRIT3,
     *IWRIT4,IWRIT5,IWRIT6,IOPTN,IWT
 4810 FORMAT(3I3,I11,I5,7I2,I2,I2)
C
C       THE INPUT  -  SECOND SET OF CARDS TO BE INPUTTED.
C
C       ITYPE(I), I=1,NC
C               WHERE ITYPE=0 IF THE ITH BETA CONSTRAINT IS < OR =
C                          =1 IF THE ITH BETA CONSTRAINT IS > OR =
C                          =2 IF THE ITH BETA CONSTRAINT IS AN EQUALITY
C
C       IF THERE ARE NO CONSTRAINTS PLACED ON BETA (NC=0), THEN THIS
C       SECOND CARD OF INPUT VALUES (ITYPE) IS SKIPPED COMPLETELY.
C       (DO NOT INSERT A BLANK CARD, HOWEVER.)
C
      IF(NC.EQ.0) GO TO 12
      READ(5,101) (ITYPE(I),I=1,NC)
  101 FORMAT(20I2)
C
C       THE INPUT  -  THIRD SET OF CARDS TO BE INPUTTED.
C           READ IN THE OBSERVATION VECTOR, Y, OF LENGTH NOBS.
C
   12 READ(5,100) (Y(J),J=1,NOBS)
  100 FORMAT(10F8.3)
C
C           READ IN THE NOBS BY IP MATRIX OF CONSTANT COEFFICIENTS, X,
C           BY ROWS.
C
      DO 102 J=1,NOBS
  102 READ(5,103) (X(J,I),I=1,IP)
  103 FORMAT(10F8.3)
C
C         THE INPUT  -  FOURTH SET OF CARDS, FOR THE BETA CONSTRAINTS.
C         IF THERE ARE NO CONSTRAINTS ON BETA, THEN THE INPUT IS NOW
C         COMPLETED.  IF THERE ARE BETA CONSTRAINTS, THESE LAST SET
C         OF INPUT CARDS ARE REQUIRED.
C
C             READ THE NC BY IP MATRIX OF CONSTANT COEFFICIENTS, A,
C             BY ROWS.
C
      INDNC=1
      IF(NC.EQ.0) GO TO 206
      DO 105 J=1,NC
  105 READ(5,104) (A(J,I),I=1,IP)
  104 FORMAT(10F8.3)
  206 CONTINUE
C
C             READ THE VECTOR OF LENGTH NC FORMING THE RIGHT HAND
C             SIDES OF THE CONSTRAINTS FOR BETA, B.
C
      IF(NC.NE.0) READ(5,244) (B(J),J=1,NC)
  244 FORMAT(10F8.3)
C
C             THE INPUT - FIFTH SET OF CARDS.
C             THE VALUES OF THE RESIDUAL WEIGHT COEFFICIENTS, WT.
C             THIS INPUT IS OPTIONAL AND IS READ ONLY WHEN IWT=1.
C
      IF(IWT.EQ.1) READ(5,2333) (WT(J),J=1,NOBS)
 2333 FORMAT(10F8.3)
C
C              USER SUPPLIED TITLE CARD READ IN.
C
      READ(5,2331) (TITLE(I),I=1,20)
 2331 FORMAT(20A4)
C
C        INPUT COMPLETED.
C
C
C        CALCULATE:
C        NT0 = THE NUMBER OF CONSTRAINTS ON BETA OF THE FORM:
C              A*BETA < OR = B.
C        NT1 = THE NUMBER OF CONSTRAINTS ON BETA OF THE FORM:
C              A*BETA > OR = B.
C        NT2 = THE NUMBER OF CONSTRAINTS ON BETA OF THE FORM:
C              A*BETA = B.
C
      IF(NC.NE.0) GO TO 19
      ITYPE(1)=2
      DO 9 J=1,IP
    9 A(1,J)=0.D0
      B(1)=0.D0
      NC=1
      INDNC=0
   19 NT0=0
      NT1=0
      NT2=0
      DO 250 I=1,NC
      IF(ITYPE(I).EQ.0) NT0=NT0+1
      IF(ITYPE(I).EQ.1) NT1=NT1+1
      IF(ITYPE(I).EQ.2) NT2=NT2+1
  250 CONTINUE
C
C        CALCULATE:
C        M = THE NUMBER OF CONSTRAINTS NOT INCLUDING THE OBJECTIVE
C            FUNCTION
C        N = THE NUMBER OF VARIABLES
C
      M=NT0 + NT1 + NT2 + NOBS
      N = 2*IP +2*NOBS +NT0 + NT1
      MP1=M+1
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      MP2=M+2
      ITERA=0
C
C           PRINTING OF HEADINGS AND INFORMATION
C
      WRITE(6,2321)
 2321 FORMAT(1H1,19X,'MR. A. :',/,/,6X,
     *'MINIMIZES THE ABSOLUTE RESIDUALS.',/,/,6X,
     *'     THIS PROGRAM ESTIMATES A LINEAR REGRESSION BY MINIMIZING',
     */,6X,'THE SUM OF THE ABSOLUTE RESIDUALS - L1 ESTIMATION.',/,
     *6X,'IN ADDITION, A MINI-MONTE CARLO SIMULATION GENERATES AN',
     */,6X,'ESTIMATED COVARIANCE MATRIX FOR THE ESTIMATED',/,6X,
     *'REGRESSION PARAMETERS.',/,6X,'     IN ADDITION, LINEAR CONSTR',
     *'AINTS MAY  ',/,6X,'BE PLACED ON THE PARAMETER VECTOR, BETA.   '
     *,/,6X,'THESE CONSTRAINTS MAY BE EQUALITY OR INEQUALITY CONTSTRAI',
     *'NTS.',
     */,6X,'     THE FOLLOWING PROCEDURE DEVELOPED BY :',/,15X,
     *'D.N. BOOK',/,15X,'J.B. BOOKER',/,15X,'H.O. HARTLEY',/,15X,
     *'R.L. SIELKEN, JR.',/,11X,'INSTITUTE OF STATISTICS',/,11X,
     *'TEXAS A & M UNIVERSTIY',/,11X,'COLLEGE STATION, TEXAS 77843',
     */,/,11X,'INQUIRIES AND COMMENTS SHOULD BE ADDRESSED TO:',/,
     *15X,'ROBERT L. SIELKEN, JR.',/,/,6X,'     THE SUPPORT OF THE ',
     *'OFFICE OF NAVAL RESEARCH IS GRATEFULLY ACKNOWLEDGED.')
C
C           PRINTOUT OF INPUTTED QUANTITIES.
C
      WRITE(6,2332) (TITLE(I),I=1,20)
 2332 FORMAT(1H1,5X,20A4,/)
      WRITE(6,2920) NOBS,IP,NSEED,ISAM,NC
 2920 FORMAT(1H0,5X,'NUMBER OF OBSERVATIONS = ',I5,/,
     *6X,'NUMBER OF PARAMETERS = ',I5,/,
     *6X,'USER SUPPLIED RANDOM INTEGER, NSEED = ',I11,
     */,6X,'SAMPLE SIZE FOR THE MINI MONTE CARLO STUDY = ',I5,/,
     *6X,'NUMBER OF BETA CONSTRAINTS = ',I5)
      IF(INDNC.EQ.1) WRITE(6,2921) NTO,NT1,NT2
 2921 FORMAT(1H0,9X,'NUMBER OF < OR = BETA CONSTRAINTS = ',I5,/,
     *1H ,9X,'NUMBER OF > OR = BETA CONSTRAINTS = ',I5,/,
     *1H ,9X,'NUMBER OF = BETA CONSTRAINTS = ',I5)
C
C        OPTIONAL PRINTOUT OF X, Y, A, AND B WHEN IWRITO=1.
C
      IF(IWRITO.EQ.0) GO TO 2305
      WRITE(6,2324)
 2324 FORMAT(1H1,5X,'THE LINEAR REGRESSION IS Y = X*BETA + EPSILON',/,
     *6X,'WHERE',/,6X,'Y CONTAINS THE OBSERVATIONS,',/,6X,
     *'BETA IS A VECTOR CONTAINING THE REGRESSION PARAMETERS,'
     *,/,6X,'THE I-TH ROW OF X CONTAINS THE COEFFICIENTS OF BETA',
     */,6X,'CORRESPONDING TO THE I-TH OBSERVATION, AND',/,6X,
     *'EPSILON IS A RANDOM VARIABLE WITH MEAN ZERO AND VARIANCE '
     *,/,6X,'SIGMA-SQUARED REPRESENTING RANDOM VARIABILITY FROM',
     */,6X,'X*BETA.')
      WRITE(6,2306)
 2306 FORMAT(1H0,10X,'THE Y VECTOR')
      DO 2307 I=1,NOBS
 2307 WRITE(6,2308) Y(I)
 2308 FORMAT(1H ,15X,F16.5)
      WRITE(6,2301)
 2301 FORMAT(1H0,10X,'THE X MATRIX')
      DO 2302 I=1,NOBS
 2302 WRITE(6,2303) (X(I,J), J=1,IP)
 2303 FORMAT(1H ,15X,16F7.1)
      WRITE(6,2309)
 2309 FORMAT(1H0,5X,' THE CONSTRAINTS PLACED ON BETA OF THE FORM',
     *' A*BETA = B',/,9X,'WHERE THE CONSTRAINTS MAY BE EQUALITY OR ',
     *'INEQUALITY CONSTRAINTS.',/,9X,'A = THE MATRIX OF COEFFICIENTS'
     *,' WHOSE I-TH ROW CORRESPONDS TO THE I-TH INEQUALITY.')
      WRITE(6,2310)
 2310 FORMAT(1H0,10X,'THE RIGHT HAND SIDES OF THE BETA CONSTRAINTS',
     *', THE VECTOR B')
      DO 2311 J=1,NC
 2311 WRITE(6,2312) B(J)
 2312 FORMAT(1H ,9X,F16.5)
      WRITE(6,2313)
 2313 FORMAT(1H0,10X,'THE COEFFICIENTS FOR THE BETA ',
     *'CONSTRAINTS, THE A MATRIX')
      DO 2314 I=1,NC
 2314 WRITE(6,2315) (A(I,J),J=1,IP)
 2315 FORMAT(1H ,9X,10F10.4)
      IF(IWT.EQ.0) GO TO 2316
      WRITE(6,2317)
 2317 FORMAT(1H0,10X,'THE RESIDUAL WEIGHT COEFFICIENTS')
      DO 2318 I=1,NOBS
 2318 WRITE(6,2319) WT(I)
 2319 FORMAT(1H ,15X,F16.5)
 2316 CONTINUE
 2305 CONTINUE
C
C          CALL SUBROUTINES BTILDE AND RHANDS TO FORMULATE BETAO AND
C          THE PROPER RIGHT HAND SIDES RESPECTIVELY.
C
      CALL BTILDE(BETAO,Y)
      CALL RHANDS(YMXB,BMAB,BETAO,Y)
C
C          FORM INDICATOR OF BMAB, IRHS.
C          IRHS = 1 IF BMAB IS POSITIVE OR ZERO
C                 2 IF BMAB IS NEGATIVE.
C
      DO 106 I=1,NC
      IF (BMAB(I).GE.0) IRHS(I) = 1
  106 IF (BMAB(I).LT.0) IRHS(I) = 2
C
C          AUGMENT THE RIGHT HAND SIDES FORMING BRHS2.
C
      BRHS2(1)=0.D0
      BRHS2(2)=0.D0
C
C          FORMULATE THE OBJECTIVE FUNCTION COEFFICIENTS, C(J),J=1,N
C
      IF(IWT.EQ.0) GO TO 117
      GO TO 116
  117 DO 114 J=1,NOBS
      WT(J)=1.D0
  114 WT(J+NOBS)=1.D0
  116 DO 13 J=1,N
   13 C(J)=0.D0
      DO 14 K=1,NOBS
      J=K+2*IP
      JJ=K+2*IP+NOBS
      C(J)=-1.D0*WT(K)
      C(JJ)=-1.D0*WT(K)
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   14 CONTINUE
C
C        FORM THE M BY NRV (NUMBER OF REAL VARIABLES) MATRIX A2
C          OF THE FORM:
C               X  -X  I  -I
C               A  -A  0   0
C
C         CALL SUBROUTINE SELECT TO SELECT PROBLEMS P1 OR P2
C         EACH WITH PROBABILITY ONE-HALF.
C
      CALL SELECT(IP1P2,ITERA)
      DO 107 I=1,NOBS
      III=I+2
      DO 108 J=1,IP
      A2(III,J)=X(I,J)
      A2(III,J+IP)=-X(I,J)
      IF(IP1P2.EQ.2) A2(III,J)=-X(I,J)
      IF(IP1P2.EQ.2) A2(III,J+IP)=X(I,J)
  108 CONTINUE
      DO 110 K=1,NOBS
      A2(III,2*IP+K)=0.D0
  110 A2(III,2*IP+NOBS+K)=0.D0
      A2(III,I+2*IP)=1.D0
      A2(III,I+2*IP+NOBS)=-1.D0
  107 CONTINUE
      DO 213 I=1,NOBS
      DO 213 K=1,NC
      KK=2+NOBS+K
      A2(KK,I+NOBS+2*IP)=0.D0
  213 A2(KK,I+2*IP)=0.D0
C
C        AUGMENTATION OF MATRIX A2 BY THE ADDITION OF THE COEFFICIENTS
C        OF THE BETA CONSTRAINTS, MATRIX A.
C
      DO 109 I=1,NC
      DO 118 J=1,IP
      A2(NOBS+I+2,J)=A(I,J)
      A2(NOBS+I+2,J+IP)=-A(I,J)
      IF(IP1P2.EQ.2) A2(NOBS+I+2,J)=-A(I,J)
      IF(IP1P2.EQ.2) A2(NOBS+I+2,J+IP)=A(I,J)
  118 CONTINUE
  109 CONTINUE
C
C       AUGMENTATION OF MATRIX A2 BY ADDING SURPLUS/SLACK VARIABLES.
C
      NRV=2*IP+2*NOBS
      KK=NT0+NT1
      IF(KK.EQ.0) GO TO 202
      DO 210 I=1,M
      DO 210 K=1,KK
      II=I+2
      J=NRV+K
      A2(II,J)=0.D0
  210 CONTINUE
      JJ=0
      DO 211 J=1,NC
      IF(ITYPE(J).EQ.2) GO TO 211
      JJ=JJ+1
      I=NRV+JJ
      K=NOBS+J+2
      A2(K,I)=1.D0
      IF(ITYPE(J).EQ.1) A2(K,I)=-1.D0
  211 CONTINUE
  202 DO 200 J=1,N
      A2(1,J)=-C(J)
  200 A2(2,J)=0.D0
C
C        FORM THE BASIS INVERSE, M+2 BY M+2 MATRIX, AND THE INITIAL
C        BASIC FEASIBLE SOLUTION.
C
      DO 17 I=1,MP2
      DO 16 L=1,MP2
   16 B2INV(L,I)=0.D0
   17 B2INV(I,I)=1.D0
C
C           CONSTRUCT THE INITIAL BASIS INVERSE, INDEX OF THE INITIAL
C             BASIC  VARIABLES, AND THE INITIAL BASIC VARIABLE VALUES.
C
      DO 240 I=1,NOBS
      IF(YMXB(I).GE.0) GO TO 241
      INBASE(2+I)=2*IP+NOBS+I
      B2INV(1,2+I)=1.D0
      B2INV(2+I,2+I)=-1.D0
      XB2(2+I)=-YMXB(I)
      BRHS2(2+I)=YMXB(I)
      GO TO 243
  241 INBASE(2+I)=2*IP+I
      B2INV(1,2+I)=-1.D0
      XB2(2+I)=YMXB(I)
      BRHS2(2+I)=YMXB(I)
  243 CONTINUE
  240 CONTINUE
      INBASE(1)=0
      INBASE(2)=50
C
C           VARIABLE 50 EQUALS MINUS THE SUM OF THE OTHER ARTIFICIAL
C           VARIABLES WHICH ARE NUMBERED AS 51,52,. . .
C
      XB2(1)=0.D0
      NOBSP2=NOBS+2
      DO 220 I=3,NOBSP2
  220 XB2(1)=XB2(1)-DABS(BRHS2(I))
      NARTS=0
      NSLACK=0
      NSURP=0
      III=2*IP+NOBS+1
      DO 235 I=1,NC
      IJ=NOBSP2+I
      BRHS2(IJ)=BMAB(I)
      IF(ITYPE(I).EQ.2) GO TO 222
      IF(ITYPE(I).EQ.1) GO TO 221
C
C           TYPE 0 A CONSTRAINT:
C
      IF(BMAB(I).LT.0) GO TO 230
      NSLACK=NSLACK+1
      INBASE(IJ)=NRV+NSLACK
      XB2(IJ)=BMAB(I)
      GO TO 235
  230 NARTS=NARTS+1
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      INBASE(IJ)=50+NARTS
      XB2(IJ)=-BMAB(I)
      B2INV(IJ,IJ)=-1.D0
      B2INV(2,IJ)=1.D0
      GO TO 235
C
C          TYPE 1 A CONSTRAINT:
C
  221 IF(BMAB(I).GT.0) GO TO 231
      NSURP=NSURP+1
      INBASE(IJ)=NRV+NTO+NSURP
      XB2(IJ)=-BMAB(I)
      B2INV(IJ,IJ)=-1.D0
      GO TO 235
  231 NARTS=NARTS+1
      INBASE(IJ)=50+NARTS
      XB2(IJ)=BMAB(I)
      B2INV(2,IJ)=-1.D0
      GO TO 235
C
C          TYPE 2 A CONSTRAINT:
C
  222 NARTS=NARTS+1
      INBASE(IJ)=50+NARTS
      XB2(IJ)=DABS(BMAB(I))
      B2INV(2,IJ)=1.D0
      B2INV(IJ,IJ)=-1.D0
  235 CONTINUE
C
C          THE SLACKS ARE NUMBERED AS (2*IP+2*NOBS)+1,...,
C                                     (2*IP+2*NOBS)+NTO.
C          THE SURPLUS VARIABLES ARE NUMBERED AS (2*IP+2*NOBS+NTO)+1,
C                   ...,(2*IP+2*NOBS+NTO)+NT1.
C
C
C          CALCULATION OF XB2(2) = - SUM OF THE ARTIFICIAL VARIABLES.
C
      IF(NARTS.EQ.0) GO TO 269
      SUMS=0.D0
      DO 267 I=1,MP2
      DINDX(I)=0.D0
  267 IF(INBASE(I).GT.50) DINDX(I)=1.D0
      DO 266 J=1,M
      JM=J+2
  266 SUMS=SUMS+XB2(JM)*DINDX(JM)
      XB2(2)=-SUMS
      GO TO 268
  269 XB2(2)=0.D0
  268 CONTINUE
C
C        OPTIONAL PRINTOUT OF BASIS INVERSE, INBASE, AND INITIAL
C        BASIC SOLUTION.
C
C
      IF(IWRIT1.EQ.0)GO TO 2000
C
C       WRITE OUT THE INITIAL PROBLEM AND THE INITIAL BASIC VARIABLES
C
      WRITE(6,500)
500   FORMAT(1H1,'     THE LINEAR PROGRAMMING PROBLEM AS IT WAS CREATED'
     *')
      WRITE(6,501)
501   FORMAT(1H0,'            THE OBJECTIVE FUNCTION COEFFICIENTS')
      DO 502 LM=1,N
502   WRITE(6,503) LM,C(LM)
503   FORMAT(1H ,15X,'C(',I3,') = ',F12.5)
      WRITE(6,504)
504   FORMAT(1H0,'            THE RIGHT-HAND SIDES')
      DO 507 LM=1,M
      LMM=LM+2
507   WRITE(6,505) LM,BRHS2(LMM)
505   FORMAT(1H ,15X,'BRHS(',I3,') = ',F12.5)
      WRITE(6,508)
508   FORMAT(1H0,'            THE CONSTRAINT MATRIX A')
      DO 509 LMM=1,MP2
509   WRITE(6,510) (A2(LMM,I),I=1,N)
510   FORMAT(1H ,15X,5(F12.5,3X))
 2000 CONTINUE
      IF(IWRIT3.EQ.1) WRITE(6,518)
  518 FORMAT(1H0,'            THE INITIAL BASIS INVERSE')
      IF(IWRIT3.EQ.0) GO TO 2022
      DO 519 I=1,MP2
  519 WRITE(6,520) I,(B2INV(I,J),J=1,MP2)
  520 FORMAT(1H ,9X,'THE',I2,' -TH ROW:',5(D18.8))
 2022 CONTINUE
      IF(IWRIT1.EQ.0)GO TO 2001
      WRITE(6,511)
511   FORMAT(1H0,'            THE VARIABLES INITIALLY IN THE AUGMENTED BAS
     *IS MATRIX')
      WRITE(6,512)(INBASE(I),I=1,MP2)
512   FORMAT(1H ,15X,25(I2,','))
      WRITE(6,513)
513   FORMAT(1H0,'            THE INITIAL VALUES OF THE BASIC VARIABLES')
      DO 515 I=1,MP2
515   WRITE(6,514) I,XB2(I)
514   FORMAT(1H ,15X,'XB(',I3,') = ',D15.5)
      WRITE(6,855) XB2(2)
855   FORMAT(1H0,10X,'THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = ',D2
     *0.10)
      IF(IWRIT2.EQ.1) WRITE(6,1041)
1041  FORMAT(1H0,10X,'PHASE 1 NOW BEGINS.')
2001  PHASE=1
      ITERA=0
350   CONTINUE
C
C          START THE BASIC SIMPLEX ALGORITHM
C
      ITERA=ITERA+1
      IF(IWRIT2.EQ.1)     WRITE(6,1051) PHASE ,ITERA
1051  FORMAT(1H1,25X,' PHASE ',I1,' :  ITERATION ',I3)
C
C          COMPUTE THE REDUCED COSTS: 'C(J) -Z(J)'
C          THE J-TH REDUCED COST IS DENOTED BY REDCOS(J)
C
      IF(IWRIT2.EQ.1)     WRITE(6,516)
516   FORMAT(1H0,'            THE REDUCED COSTS')
      IF(PHASE.EQ.1) IJKLM=2
      IF(PHASE.EQ.2) IJKLM=1
      DO 23 J=1,N
      R=0.D0
```

```
      DO 22 K=1,MP2
22    R=R+B2INV(IJKLM,K)*A2(K,J)
      REDCOS(J)=-R
23    IF(IWRIT2.EQ.1)     WRITE(6,517) J,REDCOS(J)
517   FORMAT(1H ,15X,'THE ',I3,'-TH REDUCED COST = ',D20.10)
C
C        FIND THE MAXIMUM REDUCED COST
C              RMAX = THE MAXIMUM REDUCED COST
C              IRMAX = THE SMALLEST INDEX J SUCH THAT REDCOS(J)=RMAX
C
      IRMAX=1
      RMAX=REDCOS(1)
      DO 24 J=2,N
      IF(REDCOS(J) .LE. RMAX) GO TO 24
      RMAX=REDCOS(J)
      IRMAX=J
24    CONTINUE
C
C        IF RMAX IS NOT POSITIVE THEN ALL THE REDUCED COSTS ARE
C        NON-POSITIVE AND THE CURRENT SOLUTION IS AN OPTIMAL SOLUTION
C
      IF(RMAX .LE. TOLR1) GO TO 401
      IF(IWRIT2.EQ.1)     WRITE(6,525) IRMAX
525   FORMAT(1H0,'         X(',I3,') IS THE NON-BASIC VARIABLE WITH THE
     * SMALLEST INDEX ',/,11X,'WHICH HAS THE LARGEST POSITIVE REDUCED CO
     *ST.')
      IF(IWRIT2.EQ.1)     WRITE(6,526) IRMAX
526   FORMAT(1H ,'           THEREFORE X(',I3,') WILL ENTER THE BASIS.')
C
C        DETERMINE THE IRMAX-TH Y2 VECTOR
C
      DO 26 L=1,MP2
      Y2(L)=0.D0
      DO 26 LL=1,MP2
26    Y2(L)=Y2(L)+B2INV(L,LL)*A2(LL,IRMAX)
      IF(IWRIT2.EQ.1)     WRITE(6,1031) IRMAX
1031  FORMAT(1H0,10X,'THE COMPONENTS OF Y(-,',I3,')')
      IF(IWRIT2.EQ.0)GO TO 2002
      IF(PHASE.EQ.1) IDUM=3
      IF(PHASE.EQ.2) IDUM=2
      DO 1033 I=IDUM,MP2
1033  WRITE(6,1032) I,IRMAX,Y2(I)
1032  FORMAT(1H ,15X,'Y(',I3,',',I3,') = ',D20.10)
2002  IF(PHASE.EQ.1) GO TO 1001
C
C        ARE ALL THE COMPONENTS OF Y NON-POSITIVE
C
      NUMBER=0
      DO 27 L=2,MP2
27    IF(Y2(L) .LE. TOLR2) NUMBER=NUMBER+1
C
C        IF NUMBER = M+1 THEN ALL OF THE COMPONENTS OF Y ARE
C        NON-POSITIVE AND THE PROBLEM HAS AN UNBOUNDED SOLUTION.
C
      IF(NUMBER .EQ. MP1) GO TO 402
1001  CONTINUE
C
C        THE IRMAX-TH VARIABLE IS TO ENTER THE BASIS.  (IRMAX=1,...,N+1)
C
C        THE VARIABLE TO LEAVE THE BASIS IS NOW DETERMINED.
C              RMIN = THE MINIMUM RATIO OF XB(I)/Y(I) WITH Y(I) > 0.
C               IRMIN = THE INDEX OF THE BASIS COLUMN TO BE REMOVED
C              IRMIN = 3,...,MP2 IN PHASE 1
C              IRMIN = 2,...,MP2 IN PHASE 2
C
      IF(IWRIT2.EQ.1)     WRITE(6,534) IRMAX,IRMAX
534   FORMAT(1H0,'          THE VECTOR TO LEAVE THE BASIS IS NOW DETERMI
     *NED.',/,11X,'THE XB(I)/Y(I,',I3,') RATIOS WITH Y(I,',I3,') > 0. AR
     *E:')
       RMIN=.99D 20
      IRMIN=0.
      I=4-PHASE
      DO 32 II=I,MP2
      IF(Y2(II).LE.TOLR2) GO TO 32
      R=XB2(II)/Y2(II)
      IF(IWRIT2.EQ.1)     WRITE(6,535) R,II,II,IRMAX
535   FORMAT(1H ,15X,E15.5,'= XB(',I3,')/Y(',I3,',',I3,')')
      RR=R-RMIN
      IF(RR .GE. 0.D0) GO TO 32
      RMIN=R
      IRMIN=II
32    CONTINUE
      IF(IWRIT2.EQ.1)     WRITE(6,540) IRMIN,IRMIN,IRMAX,RMIN,IRMIN
540   FORMAT(1H0,'           THE MINIMUM RATIO IS XB(',I3,')/Y(',I3,',',I
     *3,') = ',D20.10,',',/,11X,'THEREFORE THE ',I3,'-TH VARIABLE LEAVES
     * THE BASIS.')
C
C        REPLACE THE IRMIN-TH COLUMN OF THE BASIS BY THE IRMAX-TH
C        COLUMN OF A
C
C        UPDATE THE BASIS INVERSE: B2INV
C
      DO 33 J=2,MP2
      WW=B2INV(IRMIN ,J)/Y2(IRMIN )
      DO 37 L=1,MP2
37    B2INV(L,J)=B2INV(L,J)-WW*Y2(L)
33    B2INV(IRMIN ,J)=WW
C
C        UPDATE THE BASIC VARIABLES: INBASE AND XB1
C
2003  INBASE(IRMIN)=IRMAX
      W=XB2(IRMIN )/Y2(IRMIN )
      DO 38 I=1,MP2
38    XB2(I)=XB2(I)-Y2(I)*W
      XB2(IRMIN )=W
      IF(IWRIT2.EQ.0)GO TO 1050
      WRITE(6,541)
541   FORMAT(1H0,'           THE BASIC VARIABLES ARE NOW')
      WRITE(6,542) (INBASE(K),K=1,MP2)
542   FORMAT(1H ,15X,25(I2,','))
      WRITE(6,543)
543   FORMAT(1H0,10X,'THE VALUES OF THE BASIC VARIABLES ARE NOW')
      DO 544 K=1,MP2
544   WRITE(6,545) K,INBASE(K),XB2(K )
545   FORMAT(1H ,15X,'XB(',I3,') = X(',I3,') = ',D20.10)
      IF(PHASE.EQ.1) IJKLM=2
      IF(PHASE.EQ.2) IJKLM=1
      WRITE(6,952) XB2(IJKLM)
952   FORMAT(1H0,10X,'THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = ',E1
     *5.5)
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      WRITE(6,4801)                                                     MAIN0721
4801  FORMAT(1H0,10X,'THE CURRENT BASIS INVERSE IS AS FOLLOWS:')        MAIN0722
      DO 4802 I=1,MP2                                                   MAIN0723
4802  WRITE(6,4803) I,(B2INV(I,J),J=1,MP2)                              MAIN0724
4803  FORMAT(1H ,12X,'THE ',I2,'-TH ROW: ',5(D18.8,2X))                 MAIN0725
 1050 GO TO 350                                                         MAIN0726
401   IF(PHASE.EQ.2) GO TO 1010                                         MAIN0727
      SUMART=-XB2(2)                                                    MAIN0728
      IF(SUMART.LT.TOLR3) GO TO 1011                                    MAIN0729
      WRITE(6,1012)                                                     MAIN0730
1012  FORMAT(1H0,5X,'THE VALUE OF THE SUM OF THE ARTIFICIAL VARIABLES ATMAIN0731
     * THE END OF PHASE 1',/,11X,'IS NON-ZERO.  HENCE THE GIVEN LP PROBLMAIN0732
     *EM IS INFEASIBLE.')                                               MAIN0733
      WRITE(6,853)                                                      MAIN0734
  853 FORMAT(1H1)                                                       MAIN0735
      GO TO 999                                                         MAIN0736
1011  IF(IWRIT2.EQ.0)GO TO 2004                                         MAIN0737
      WRITE(6,1013) SUMART                                              MAIN0738
1013  FORMAT(1H0,10X,                                                   MAIN0739
     *                'THE VALUE OF THE SUM OF THE ARTIFICIAL VARIABLES ATMAIN0740
     * THE END OF PHASE 1',/,11X,'IS ZERO.  (ACTUALLY IT IS ',D20.10,')'MAIN0741
     *,//,15X,'PHASE 2 NOW BEGINS.',//)                                 MAIN0742
      WRITE(6,1040)                                                     MAIN0743
1040  FORMAT(1H ,10X,'THE INITIAL FEASIBLE SOLUTION IS AS FOLLOWS:')    MAIN0744
      DO 5820 J=1,N                                                     MAIN0745
5820  XSOL(J) = 0.D0                                                    MAIN0746
      DO 1015 LM = 2,MP2                                                MAIN0747
      K = INBASE(LM)                                                    MAIN0748
1015  XSOL(K) = XB2(LM)                                                 MAIN0749
      DO 5281 J=1,N                                                     MAIN0750
5281  WRITE(6,523) J,XSOL(J)                                            MAIN0751
      WRITE(6,852) XB2(1)                                               MAIN0752
2004  PHASE=2                                                           MAIN0753
      ITERA=0                                                           MAIN0754
      GO TO 350                                                         MAIN0755
1010  IF(IWRIT2.EQ.1)  WRITE(6,531)                                     MAIN0756
531   FORMAT(1H0,10X, 'ALL OF THE REDUCED COSTS ARE NON-POSITIVE.',/,11XMAIN0757
     *,'THEREFORE THE CURRENT BASIC FEASIBLE SOLUTION IS AN OPTIMAL SOLUMAIN0758
     *TION.')                                                           MAIN0759
      IF(IWRIT1.EQ.1)  WRITE(6,521)                                     MAIN0760
521   FORMAT(1H0,10X,'THE OPTIMAL SOLUTION IS AS FOLLOWS:')             MAIN0761
      DO 4820 J=1,N                                                     MAIN0762
4820  XSOL(J) = 0.D0                                                    MAIN0763
      DO 522 LM=2,MP2                                                   MAIN0764
      K=INBASE(LM)                                                      MAIN0765
522   XSOL(K) = XB2(LM)                                                 MAIN0766
      DO 4281 J=1,N                                                     MAIN0767
4281  IF(IWRIT1.EQ.1) WRITE(6,523) J,XSOL(J)                            MAIN0768
523   FORMAT(1H ,15X,'X(',I3,') = ',D20.10)                             MAIN0769
      IF(IWRIT1.EQ.1) WRITE(6,851) XB2(1)                               MAIN0770
851   FORMAT(1H0,10X, 'THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS ',MAIN0771
     * E15.5)                                                           MAIN0772
C                                                                       MAIN0773
C                                                                       MAIN0774
C        FORMULATE THE L1 ESTIMATE AND OUTPUT.                          MAIN0775
C                                                                       MAIN0776
C                                                                       MAIN0777
      IF(IP1P2.EQ.2) GO TO 290                                          MAIN0778
      DO 280 J=1,IP                                                     MAIN0779
  280 BHAT(J)=XSOL(J)-XSOL(J+IP)+BETA0(J)                               MAIN0780
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      GO TO 291                                                         MAIN0781
  290 DO 292 J=1,IP                                                     MAIN0782
  292 BHAT(J)=XSOL(J+IP)-XSOL(J)+BETA0(J)                               MAIN0783
  291 CONTINUE                                                          MAIN0784
      IF(IWRIT0.EQ.1.OR.IWRIT1.EQ.1.OR.IWRIT2.EQ.1.OR.IWRIT3.EQ.1.OR.   MAIN0785
     *IWRIT4.EQ.1.OR.IWRIT5.EQ.1.OR.IWRIT6.EQ.1) WRITE(6,2980)          MAIN0786
 2980 FORMAT(1H1)                                                       MAIN0787
      WRITE(6,2900)                                                     MAIN0788
 2900 FORMAT(1H0,4X,' THE AUXILIARY LEAST SQUARES ESTIMATE, BETA0,',    MAIN0789
     *' OF THE REGRESSION PARAMETER VECTOR, BETA')                      MAIN0790
      DO 2901 J=1,IP                                                    MAIN0791
 2901 WRITE(6,2902) J,BETA0(J)                                          MAIN0792
 2902 FORMAT(1H ,12X,'LEAST SQUARES ESTIMATE OF BETA(',I3,') = ',F14.6) MAIN0793
      WRITE(6,2903)                                                     MAIN0794
 2903 FORMAT(1H0,5X,'MR.A',1H','S ANSWER :',12X,'THE ESTIMATE OF THE ', MAIN0795
     *'REGRESSION PARAMETER VECTOR WHICH MINIMIZES THE SUM OF THE',     MAIN0796
     *' ABSOLUTE RESIDUALS')                                            MAIN0797
      DO 2904 J=1,IP                                                    MAIN0798
 2904 WRITE(6,2905) J,BHAT(J)                                           MAIN0799
 2905 FORMAT(1H ,12X,'L1 ESTIMATE OF BETA(',I3,') = ',F14.6)            MAIN0800
      OPTOBJ=DABS(XB2(1))                                               MAIN0801
C                                                                       MAIN0802
C        PRINTOUT OF THE RESIDUALS.                                     MAIN0803
C                                                                       MAIN0804
      DO 260 I=1,NOBS                                                   MAIN0805
      K=2*IP+I                                                          MAIN0806
      RES1(I)=XSOL(K)                                                   MAIN0807
      RES2(I)=XSOL(K+NOBS)                                              MAIN0808
  260 RES(I)=RES1(I)-RES2(I)                                            MAIN0809
      SUMRES=0.D0                                                       MAIN0810
      DO 261 LL=1,NOBS                                                  MAIN0811
  261 SUMRES=SUMRES+DABS(RES(LL))                                       MAIN0812
      WRITE(6,254)                                                      MAIN0813
  254 FORMAT(1H0,5X,'THE RESIDUALS : ')                                 MAIN0814
      DO 253 I=1,NOBS                                                   MAIN0815
  253 WRITE(6,255) I,RES(I)                                             MAIN0816
  255 FORMAT(1H ,12X,'THE ',I3,'-TH RESIDUAL = ',F14.6)                 MAIN0817
      WRITE(6,356) SUMRES                                               MAIN0818
  356 FORMAT(1H0,5X,'THE SUM OF THE ABSOLUTE RESIDUALS = ',F14.6)       MAIN0819
      RESMAX=0.D0                                                       MAIN0820
      DO 357 J=1,NOBS                                                   MAIN0821
      IF(RESMAX.LE.DABS(RES(J))) RESMAX=DABS(RES(J))                    MAIN0822
  357 CONTINUE                                                          MAIN0823
      WRITE(6,358) RESMAX                                               MAIN0824
  358 FORMAT(1H0,5X,'THE MAXIMUM ABSOLUTE RESIDUAL = ',F14.6)           MAIN0825
C                                                                       MAIN0826
C                                                                       MAIN0827
C       LOOP WHICH GENERATES THE MINI-MONTE CARLO STUDY.                MAIN0828
C                                                                       MAIN0829
C                                                                       MAIN0830
      NOUP12=IP1P2                                                      MAIN0831
      SUMOBJ=0.D0                                                       MAIN0832
      IF(IWRIT3.EQ.1.OR.IWRIT4.EQ.1) WRITE(6,2910)                      MAIN0833
 2910 FORMAT(1H1,'THE MONTE CARLO STUDY TO ESTIMATE THE COVARIANCE OF', MAIN0834
     *' THE L1 ESTIMATE OF BETA NOW BEGINS',/,/)                        MAIN0835
      DO 2222 KKK=1,ISAM                                                MAIN0836
      IF(IWRIT4.EQ.1) WRITE(6,2911) KKK                                 MAIN0837
 2911 FORMAT(1H ,9X,'RESULTS FOR MONTE CARLO SAMPLE NUMBER ',I3,':')    MAIN0838
C                                                                       MAIN0839
C       CALL APPROPRIATE SUBROUTINE TO GENERATE EPSILONS AS EITHER      MAIN0840
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C       NORMAL (NORMAL), UNIFORM (UNIFRM) OR DOUBLE EXPONENTIAL (DOUBLE)
C
      IF(IOPTN.EQ.1) CALL NORMAL(NOBS,ESTAR)
      IF(IOPTN.EQ.2) CALL DOUBLE(NOBS,ESTAR)
      IF(IOPTN.EQ.3) CALL UNIFRM(NOBS,ESTAR)
      ITERA=KKK
      AK=KKK
      P=IP
      AN=NOBS
      IF(ITERA.EQ.1) GO TO 310
      SIGMA1=OPTOBJ/(SUMOBJ/AK)
      GO TO 311
  310 SIGMA1=OPTOBJ/(AN-P)
  311 CONTINUE
C
C       FORM THE INDICATOR ISTAT TO USE IN DUAL SIMPLEX.
C
      DO 281 J=1,N
      ISTAT(J)=0
      DO 282 K=1,MP2
      IF(INBASE(K).EQ.50) ISTAT(50)=1
  282 IF(INBASE(K).EQ.J) ISTAT(J)=1
  281 CONTINUE
C
C       CALL THE SUBROUTINE TO SELECT PROBLEM P1 OR P2 TO SOLVE.
C
      CALL SELECT(IP1P2,ITERA)
      IF(IP1P2.EQ.NOWP12) GO TO 343
      DO 345 J=1,MP2
      DO 345 K=1,IP
      L = K+IP
      IF(INBASE(J).EQ.K) GO TO 344
      IF(INBASE(J).EQ.L) GO TO 346
      GO TO 345
  344 INBASE(J)=L
      ISTAT(K)=0
      ISTAT(L)=1
      GO TO 345
  346 INBASE(J)=L-IP
      ISTAT(L)=0
      ISTAT(L-IP)=1
  345 CONTINUE
  343 CONTINUE
      IF(IP1P2.EQ.1) GO TO 312
      DO 313 I=1,NOBS
      II=I+2
      DO 313 J=1,IP
      A2(II,J)=-X(I,J)
      A2(II,J+IP)=X(I,J)
  313 CONTINUE
      DO 314 I=1,NC
      DO 314 J=1,IP
      A2(NOBS+I+2,J)=-A(I,J)
  314 A2(NOBS+I+2,J+IP)=A(I,J)
      GO TO 111
  312 CONTINUE
      DO 112 I=1,NOBS
      II=I+2
      DO 112 J=1,IP
      A2(II,J)=X(I,J)
      A2(II,J+IP)=-X(I,J)
  112 CONTINUE
      DO 113 I=1,NC
      DO 113 J=1,IP
      A2(NOBS+I+2,J)=A(I,J)
      A2(NOBS+I+2,J+IP)=-A(I,J)
  113 CONTINUE
  111 CONTINUE
C
C       CALL SUBROUTINES TO GENERATE BETA0 FOR THE MMC STUDY AND TO
C       SOLVE THE CURRENT SAMPLE USING THE DUAL SIMPLEX ALGORITHM.
C
      CALL BTILDE(DBETA0,ESTAR)
C
C       CALL SUBROUTINE MCRHS TO FORM THE RIGHT HAND SIDES IN THE
C       MINI MONTE CARLO STUDY.
C
      CALL MCRHS(ESTAR,SIGMA1,BHAT,DBETA0,BRHS1,IP1P2)
C
C        CALCULATE XB1, THE INITIAL BASIC VARIABLES FOR USE IN THE
C        DUAL SIMPLEX ALGORITHM IN THE MINI MONTE CARLO STUDY.
C
      DO 283 I=1,MP2
      ADD = 0.D0
      DO 284 J=1,MP2
  284 ADD=ADD+B2INV(I,J)*BRHS1(J)
  283 XB1(I)=ADD
C
C       CALL THE DUAL SIMPLEX ALGORITHM TO OBTAIN THE SOLUTION,
C       DELTA BETA STAR, FOR THIS SAMPLE.
C
      CALL DUALS(B2INV,A2,INBASE,BRHS1,ISTAT,DBX,XB1)
      SUMOBJ=SUMOBJ-XB1(1)
      IF(IP1P2.EQ.1) GO TO 315
      DO 316 L=1,IP
      DB2(L)=DBX(L)
      DB1(L)=DBX(L+IP)
  316 DELB(L,ITERA)=DB1(L)-DB2(L)+DBETA0(L)
      GO TO 317
  315 DO 318 L=1,IP
      DB1(L)=DBX(L)
      DB2(L)=DBX(L+IP)
  318 DELB(L,ITERA)=DB1(L)-DB2(L)+DBETA0(L)
  317 CONTINUE
      NOWP12=IP1P2
 2222 CONTINUE
C
C       CALCULATE THE COVARIANCE OF DELTA BETA STAR BASED ON THE
C       SAMPLES OF EPSILONS FROM THE MINI MONTE CARLO STUDY.
C
      SAM=ISAM
      DO 319 J=1,IP
      SMM=0.D0
      DO 320 L=1,ISAM
  320 SMM=SMM+DELB(J,L)
  319 SUM(J)=SMM
      DO 321 I=1,IP
      DO 321 J=1,IP
      SUMSQ=0.D0
      SQSUM=0.D0
```

```
      SQSUM=SUM(J)*SUM(I)
      DO 322 L=1,ISAM
      SUMSQ=SUMSQ+DELB(J,L)*DELB(I,L)
  322 CONTINUE
      VARDBS(I,J)=(SUMSQ-SQSUM/SAM)/(SAM-P)
  321 CONTINUE
      IF(IWRIT6.EQ.1) WRITE(6,330)
  330 FORMAT(1H1,5X,'AUXILIARY RESULTS OF THE MINI MONTE CARLO STUDY',/,
     */,9X,'VALUES OF DELTA BETA STAR')
      IF(IWRIT6.EQ.0) GO TO 335
      DO 331 I=1,ISAM
  331 WRITE(6,332) I,(DELB(L,I),L=1,IP)
  332 FORMAT(1H ,9X,'     SAMPLE NUMBER =',I3,3X,10F11.5)
      WRITE(6,333)
  333 FORMAT(1H0,8X,'ESTIMATED COVARIANCE OF DELTA BETA STAR')
      DO 334 I=1,IP
  334 WRITE(6,336) (VARDBS(I,J),J=1,IP)
  336 FORMAT(1H ,12X,8F14.6)
  335 CONTINUE
C
C       CALCULATE THE ESTIMATE OF SIGMA, SIGMA HAT.
C
      SIGHAT=OPTOBJ/(SUMOBJ/SAM)
      IF(IWRIT6.EQ.1) WRITE(6,337) SUMOBJ
  337 FORMAT(1H0,'          SUM OF THE OPTIMAL OBJECTIVE FUNCTIONS OVER',
     *' ALL SAMPLES =',F14.6)
      WRITE(6,338) SIGHAT
  338 FORMAT(1H1,5X,'MAIN RESULTS OF THE MINI MONTE CARLO STUDY',/,/,6X,
     *'ESTIMATED VALUE OF SIGMA =',F14.6)
C
C        CALCULATION OF SIGMA HAT 5 AND OPTIONAL PRINTOUT WHEN IWRIT6=1.
C
      SN=NOBS
      P=IP
      IF(IOPTN.EQ.1) CCC=2.506628275D0/2.D0
      IF(IOPTN.EQ.2) CCC=DSQRT(2.D0)
      IF(IOPTN.EQ.3) CCC=1.15470053800
      SIGMA5=OPTOBJ*CCC/(SN-P)
      IF(IWRIT6.EQ.1) WRITE(6,329) SIGMA5
  329 FORMAT(1H0,5X,'AUXILIARY RESULT : THE ESTIMATE OF SIGMA 5 = ',F14.
     *6)
C
C       CALCULATION OF THE COVARIANCE OF DELTA BETA (= COV. OF BETA)
C
      DO 339 I=1,IP
      DO 339 J=1,IP
  339 VARB(I,J)=(SIGHAT**2)*VARDBS(I,J)
      WRITE(6,340)
  340 FORMAT(1H0,5X,'ESTIMATED COVARIANCE OF THE REGRESSION PARAMETER',
     *' VECTOR, BETA')
      DO 342 I=1,IP
  342 WRITE(6,341) (VARB(I,J),J=1,IP)
  341 FORMAT(1H ,11X,8F14.6)
      WRITE(6,850)
850   FORMAT(1H1)
      GO TO 999
402   WRITE(6,530)
530   FORMAT(1H0,10X, 'ALL OF THE YS ARE NON-POSITIVE.',/,11X,'HENCE THE
     * PROBLEM HAS AN UNBOUNDED SOLUTION.')
      WRITE(6,854)
  854 FORMAT(1H1)
999   CONTINUE
      STOP
      END
      SUBROUTINE DUALS(B1INV,A1,INBASE,BRHS1,ISTAT,DBX,XB1)
C
C        SUBROUTINE DUALS - THE DUAL SIMPLEX ALGORITHM  .  .  .
C
C         THE LINEAR PROGRAMMING PROBLEM IS PUT INTO THE FORM
C                MAX CX
C            SUBJECT TO
C                AX = BRHS
C                X GREATER THAN OR EQUAL TO 0
C         WHERE
C                BRHS IS A COLUMN OF CONSTANTS
C                A IS AN M-BY-N MATRIX OF CONSTANTS
C
C         THE AUGMENTED VERSIONS OF A,BRHS,B,XB,AND Y ARE REFERRED TO AS
C         A1,BRHS1,B1,XB1,AND Y1 RESPECTIVELY.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/VALUE/A(20,10),X(20,10),B(20),MP2,N,NC,NOBS,IP
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6
      DIMENSION B1INV(42,42),A1(42,72),ISTAT(135),INBASE(42),BRHS1(42)
      DIMENSION DBX(135),XB1(42),REDCOS(72),YR(72),Y1(42)
C
C        DIMENSIONES ARE: B1INV(MP2,MP2),A1(MP2,N),ISTAT(N),INBASE(MP2)
C        BRHS1(M+2),DBX(N),XB1(M+2),REDCOS(N),Y1(MP2),YR(N)
C
      MP1=MP2
      M=MP2-1
      TOLR1=1.0D-07
      TOLR2=-1.0D-07
      TOLR3=-1.0D-07
      IF(IWRIT4.EQ.0) GO TO 3555
      WRITE(6,513)
513   FORMAT(1H0,'           THE INITIAL VALUES OF THE BASIC VARIABLES')
      DO 515 I=1,MP1
      II=I-1
515   WRITE(6,514) II,XB1(I)
514   FORMAT(1H ,15X,'XB(',I3,') = ',E15.5)
 3555 CONTINUE
      IF(IWRIT4.EQ.1) WRITE(6,855) XB1(1)
855   FORMAT(1H0,10X,'THE INITIAL VALUE OF THE OBJECTIVE FUNCTION = ',E1
     *5.5)
350   CONTINUE
C
C         START THE DUAL SIMPLEX ALGORITHM
C
C
C         COMPUTE THE REDUCED COSTS:  C(J) -Z(J)
C         THE J-TH REDUCED COST IS DENOTED BY REDCOS(J)
C
      DO 23 J=1,N
      R=0.D0
      IF(ISTAT(J) .EQ. 1) GO TO 1021
      DO 22 K=1,MP1
22    R=R+B1INV(1,K)*A1(K,J)
1021  REDCOS(J)=-R
23    CONTINUE
```

```
      IF(IWRIT5.EQ.0) GO TO 2330                                        DUAL0057
      WRITE(6,1030)                                                     DUAL0058
 1030 FORMAT(1H0,'          THE REDUCED COSTS')                         DUAL0059
      DO 1031 J=1,N                                                     DUAL0060
 1031 WRITE(6,1032) J,REDCOS(J)                                         DUAL0061
 1032 FORMAT(1H ,15X,'THE ',I3,'-TH REDUCED COST = ',E15.5)             DUAL0062
 2330 CONTINUE                                                          DUAL0063
C                                                                       DUAL0064
C        FIND THE MAXIMUM REDUCED COST                                  DUAL0065
C             RMAX = THE MAXIMUM REDUCED COST                           DUAL0066
C                                                                       DUAL0067
      RMAX=REDCOS(1)                                                    DUAL0068
      DO 24 J=2,N                                                       DUAL0069
      IF(REDCOS(J) .LE. RMAX) GO TO 24                                  DUAL0070
      RMAX=REDCOS(J)                                                    DUAL0071
 24   CONTINUE                                                          DUAL0072
C                                                                       DUAL0073
C        IF RMAX IS >OR= TOLR1  THEN THE OPTIMALITY CHARACTERISTIC IS   DUAL0074
C        NOT PRESENT                                                    DUAL0075
C                                                                       DUAL0076
      IF(RMAX .GE. TOLR1)   GO TO 402                                   DUAL0077
C                                                                       DUAL0078
C        IS THE CURRENT SOLUTION FEASIBLE?                              DUAL0079
C        DETERMINE THE SMALLEST BASIC VARIABLE VALUE                    DUAL0080
C             IRMIN= THE INDEX OF THE BASIC VARIABLE WITH THE SMALLEST VDUAL0081
C             (THE INDEX IRMIN INCLUDES THE OBJECTIVE FUNCTION AS NUMBERDUAL0082
C                                                                       DUAL0083
      IRMIN=0                                                           DUAL0084
      RMIN=0.D0                                                         DUAL0085
      DO 1001 I=2,MP1                                                   DUAL0086
      W=XB1(I)-RMIN                                                     DUAL0087
      IF(W .GE. TOLR3)    GO TO 1001                                    DUAL0088
      RMIN=XB1(I)                                                       DUAL0089
      IRMIN=I                                                           DUAL0090
 1001 CONTINUE                                                          DUAL0091
C                                                                       DUAL0092
C        IF RMIN EXCEEDS TOLR3   THEN THE CURRENT SOLUTION IS FEASIBLE  DUAL0093
C        AND CONSEQUENTLY OPTIMAL                                       DUAL0094
C                                                                       DUAL0095
      IF(RMIN .GT.TOLR3)    GO TO 401                                   DUAL0096
C                                                                       DUAL0097
C        THE CURRENT SOLUTION IS STILL INFEASIBLE                       DUAL0098
C        THE IRMIN-TH VARIABLE IS TO LEAVE THE BASIS                    DUAL0099
C                                                                       DUAL0100
      IF (IWRIT5.EQ.1) WRITE(6,1040) IRMIN                              DUAL0101
 1040 FORMAT(1H0, 10X,'TENTATIVELY THE ',I3,'-TH BASIC VARIABLE IS LEAVIDUAL0102
     *NG THE BASIS')                                                    DUAL0103
C                                                                       DUAL0104
C        IS THE PROBLEM ITSELF INFEASIBLE?                              DUAL0105
C                                                                       DUAL0106
      DO 26 J=1,N                                                       DUAL0107
      YR(J)=0.D0                                                        DUAL0108
      IF(ISTAT(J) .EQ. 1) GO TO 26                                      DUAL0109
      DO 1002 LL=2,MP1                                                  DUAL0110
 1002 YR(J)=YR(J)+B1INV(IRMIN,LL)*A1(LL,J)                              DUAL0111
 26   CONTINUE                                                          DUAL0112
      IF(IWRIT5.EQ.0) GO TO 2340                                        DUAL0113
      WRITE(6,1036)                                                     DUAL0114
 1036 FORMAT(1H0,10X,'THE YRJ''S',     /,11X,'(  YR(J) = 0 IF THE J-TH VDUAL0115
     *ARIABLE IS IN THE BASIS.',/,11X,'IT CAN BE ZERO OTHERWISE TOO.)') DUAL0116
      DO 1037 J=1,N                                                     DUAL0117
 1037 WRITE(6,1038) J,YR(J)                                             DUAL0118
 1038 FORMAT(1H ,15X,'YR(',I3,') = ',E15.5)                             DUAL0119
 2340 CONTINUE                                                          DUAL0120
C                                                                       DUAL0121
C        IS THE IRMAX-TH COMPONENT OF Y NON-NEGATIVE FOR ALL OF THE     DUAL0122
C        NON-BASIC VARIABLES?                                           DUAL0123
C                                                                       DUAL0124
      NUMBER = 0                                                        DUAL0125
      DO 27 L=1,N                                                       DUAL0126
 27   IF(YR(L) .GE. TOLR2)    NUMBER=NUMBER+1                           DUAL0127
C                                                                       DUAL0128
C     IF NUMBER = N, THEN THE IRMAX-TH COMPONENT OF Y IS NON-NEGATIVE   DUAL0129
C        FOR ALL OF THE NON-BASIC VARIABLES.                            DUAL0130
C        HENCE THE PROBLEM HAS NO FEASIBLE SOLUTION.                    DUAL0131
C                                                                       DUAL0132
      IF(NUMBER .EQ. N) GO TO 403                                       DUAL0133
C                                                                       DUAL0134
C        NO INDICATION OF PROBLEM INFEASIBILITY HAS BEEN FOUND THUSFAR  DUAL0135
C        THE VARIABLE TO ENTER THE BASIS IS NOW DETERMINED              DUAL0136
C                                                                       DUAL0137
      IF (IWRIT5.EQ.1) WRITE(6,1045)                                    DUAL0138
 1045 FORMAT(1H0,10X,'THE RATIO OF THE NET PRICES TO THE NEGATIVE YRJ''SDUAL0139
     *')                                                                DUAL0140
      IRMAX=0                                                           DUAL0141
      THETA=-1.D 25                                                     DUAL0142
      DO 1004 J=1,N                                                     DUAL0143
      IF(ISTAT(J) .EQ. 1) GO TO 1004                                    DUAL0144
      IF(YR(J) .GE. TOLR2)   GO TO 1004                                 DUAL0145
      W=-REDCOS(J)/YR(J)                                                DUAL0146
      IF (IWRIT5.EQ.1) WRITE(6,1048) J,J,W                              DUAL0147
 1048 FORMAT(1H ,15X,'THE RATIO OF THE ',I3,'-TH NET PRICE TO YR(',I3,')DUAL0148
     * = ',E15.5)                                                       DUAL0149
      WW=W-THETA                                                        DUAL0150
      IF(WW .LT. 0.D0) GO TO 1004                                       DUAL0151
      THETA=W                                                           DUAL0152
      IRMAX=J                                                           DUAL0153
 1004 CONTINUE                                                          DUAL0154
C                                                                       DUAL0155
C        IRMAX = THE INDEX OF THE VARIABLE TO ENTER THE BASIS           DUAL0156
C                                                                       DUAL0157
      IF (IWRIT5.EQ.1) WRITE(6,1005) IRMIN,IRMAX                        DUAL0158
 1005 FORMAT(1H0,10X,'THE ',I3,'-TH VARIABLE IS LEAVING THE BASIS.',/,11DUAL0159
     *X,'THE ',I3,'-TH VARIABLE IS ENTERING THE BASIS.')                DUAL0160
C                                                                       DUAL0161
C        UPDATE THE BASIS INVERSE: B1INV                                DUAL0162
C                                                                       DUAL0163
C                                                                       DUAL0164
C        COMPUTE THE Y COEFFICIENTS FOR THE INCOMING VARIABLE           DUAL0165
C                                                                       DUAL0166
      DO 1025 I=1,MP1                                                   DUAL0167
      Y1(I)=0.D0                                                        DUAL0168
      DO 1025 L=1,MP1                                                   DUAL0169
 1025 Y1(I)=Y1(I)+B1INV(I,L)*A1(L,IRMAX)                                DUAL0170
C                                                                       DUAL0171
C        REPLACE THE IRMIN-TH COLUMN OF THE BASIS BY THE IRMAX-TH       DUAL0172
C        COLUMN OF A                                                    DUAL0173
C                                                                       DUAL0174
      IRMIN1=IRMIN                                                      DUAL0175
      DO 33 J=2,MP1                                                     DUAL0176
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      WW=B1INV(IRMIN1,J)/Y1(IRMIN1)
      DO 37 L=1,MP1
37    B1INV(L,J)=B1INV(L,J)-WW*Y1(L)
33    B1INV(IRMIN1,J)=WW
C
C        UPDATE THE BASIC VARIABLES: INBASE, XB1, AND ISTAT
C
C
      II=INBASE(IRMIN)
      ISTAT(II)=0
      ISTAT(IRMAX)=1
      INBASE(IRMIN)=IRMAX
      W=XB1(IRMIN1)/Y1(IRMIN1)
      DO 38 I=1,MP1
38    XB1(I)=XB1(I)-Y1(I)*W
      XB1(IRMIN1)=W
      IF(IWRIT5.EQ.0) GO TO 4805
      WRITE(6,4801)
4801  FORMAT(1H0,10X,'THE CURRENT BASIS INVERSE IS AS FOLLOWS:')
      DO 4802 I=1,MP2
4802  WRITE(6,4803) I,(B1INV(I,J),J=1,MP2)
4803  FORMAT(1H ,12X,'THE ',I2,'-TH ROW: ',5(D18.8,2X))
 4805 CONTINUE
      IF (IWRIT5.EQ.1) WRITE(6,541)
541   FORMAT(1H0,'          THE BASIC VARIABLES ARE NOW')
      IF (IWRIT5.EQ.1) WRITE(6,542) (INBASE(K),K=1,MP1)
542   FORMAT(1H ,15X,30(I3,', '))
      IF (IWRIT5.EQ.1) WRITE(6,543)
543   FORMAT(1H0,10X,'THE VALUES OF THE BASIC VARIABLES ARE NOW')
      IF (IWRIT5.EQ.0) GO TO 2350
      DO 544 K=1,M
      KK=K+1
544   WRITE(6,545) K,XB1(KK)
545   FORMAT(1H ,15X,'XB(',I3,') = ',E15.5)
      WRITE(6,852) XB1(1)
852   FORMAT(1H0,10X,'THE CURRENT VALUE OF THE OBJECTIVE FUNCTION = ',E1
     *5.5)
 2350 CONTINUE
      GO TO 350
401   IF(IWRIT4.EQ.1) WRITE(6,520)
520   FORMAT(1H0,5X,'THE CURRENT BASIC SOLUTION IS FEASIBLE AND HENCE OP
     *TIMAL.')
      IF(IWRIT4.EQ.1) WRITE(6,521)
521   FORMAT(1H ,5X,'THE NONZERO VARIABLES ARE AS FOLLOWS:')
      IF(IWRIT4.EQ.0) GO TO 2360
      DO 522 LM=1,MP2
      LMM=LM
      K=INBASE(LM)
522   WRITE(6,523)  K,XB1(LMM)
523   FORMAT(1H ,15X,'X(',I3,') = ',E15.5)
 2360 CONTINUE
      IF(IWRIT4.EQ.1) WRITE(6,851) XB1(1)
851   FORMAT(1H0,'     THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS ',
     *E15.5)
      WRITE(6,850)
850   FORMAT(1H1)
      GO TO 999
403   WRITE(6,530)
530   FORMAT(1H0,5X,'ALL OF THE YRJS ARE NON-NEGATIVE.',/,11X,'HENCE THE
     * PROBLEM HAS NO FEASIBLE SOLUTION.')
      IF(IWRIT5.EQ.1) WRITE(6,856)
856   FORMAT(1H1)
      GO TO 999
402   WRITE(6,1010)
1010  FORMAT(1H0,5X,'ALL OF THE REDUCED COSTS ARE NOT NON-POSITIVE.',/,1
     *1X,'HENCE THE DUAL SIMPLEX ALGORITHM IS NOT APPLICABLE.')
      WRITE(6,857)
857   FORMAT(1H1)
999   CONTINUE
      DO 2599 J=1,N
2599  DBX(J)=0.D0
      DO 2600 J=3,MP2
      L=INBASE(J)
2600  DBX(L)=XB1(J)
      RETURN
      END
      SUBROUTINE MCRHS(ESTAR,SIGMA1,BHAT,BETA0,BRHS1,IP1P2)
C
C
C        SUBROUTINE TO FORM THE RIGHT HAND SIDES NEEDED IN THE
C        MINI MONTE CARLO STUDY.
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/VALUE/A(20,10),X(20,10),B(20),MP2,N,NC,NOBS,IP
      COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6
      DIMENSION ESTAR(20),BHAT(10),BETA0(10),BRHS1(42)
C
C          DIMENSIONED QUANTITIES ARE: ESTAR(NOBS),BHAT(IP),BETA0(IP)
C          BRHS1(MP2)
C
      BRHS1(1)=0.D0
      BRHS1(2)=0.D0
90    DO 11 I=1,NOBS
      II=I+2
      SUM=0.D0
      DO 21 J=1,IP
21    SUM=SUM+X(I,J)*BETA0(J)
11    BRHS1(II)=ESTAR(I)-SUM
      DO 31 I=1,NC
      SSUM=0.D0
      DO 41 J=1,IP
      II=I+2
41    SSUM=SSUM+A(I,J)*BETA0(J)
31    BRHS1(II+NOBS)=(B(I)-SSUM)/SIGMA1
      M=MP2-1
      IF(IWRIT4.EQ.1) WRITE(6,50)
50    FORMAT(1H0,9X,'THE RIGHT HAND SIDES FOR THIS SAMPLE')
      IF(IWRIT4.EQ.0) GO TO 65
      DO 55 I=1,MP2
55    WRITE(6,60) I,BRHS1(I)
60    FORMAT(1H ,12X,'BRHS1(',I3,') = ',F14.6)
      WRITE(6,66)
66    FORMAT(1H ,9X,'THE LEAST SQUARES ESTIMATE OF BETA IN THIS SAMPLE')
      DO 68 J=1,IP
68    WRITE(6,69) J,BETA0(J)
69    FORMAT(1H ,12X,'BETA(',I3,') = ',F14.6)
      WRITE(6,67)
67    FORMAT(1H ,9X,'THE EPSILONS GENERATED FOR THIS SAMPLE')
      DO 71 K=1,NOBS
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 71 WRITE(6,72) K,ESTAR(K)                                          MCRH0045
 72 FORMAT(1H ,12X,'EPSILON(',I3,') = ',F14.6)                      MCRH0046
 65 CONTINUE                                                        MCRH0047
    RETURN                                                          MCRH0048
    END                                                             MCRH0049
    SUBROUTINE SELECT(IP1P2,ITERA)                                  SELT0001
C                                                                   SELT0002
C                                                                   SELT0003
C     SUBROUTINE TO SELECT EITHER PROBLEM P1 OR P2 EACH WITH        SELT0004
C     A PROBABILITY OF ONE-HALF                                     SELT0005
C                                                                   SELT0006
C                                                                   SELT0007
    DOUBLE PRECISION U                                              SELT0008
    COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6         SELT0009
    COMMON/NFEED/NSEED                                              SELT0010
    IF(IWRIT4.EQ.0) IWRIT1=0                                        SELT0011
    NSEED=NSEED+ITERA*51268                                         SELT0012
    CALL RAND(U)                                                    SELT0013
    IP1P2=1                                                         SELT0014
    IF(U.GT.5.D-1) IP1P2=2                                          SELT0015
    IF(IWRIT1.EQ.1) WRITE(6,50) IP1P2                               SELT0016
 50 FORMAT(1H0,'          THE PROBLEM P',I1,' HAS BEEN SELECTED')   SELT0017
    RETURN                                                          SELT0018
    END                                                             SELT0019
    SUBROUTINE RAND(U)                                              RAND0001
C                                                                   RAND0002
C                                                                   RAND0003
C     SUBROUTINE RAND GENERATES RANDOM UNIFORM VARIABLES.           RAND0004
C                                                                   RAND0005
C                                                                   RAND0006
    IMPLICIT REAL*8 (A-H,O-Z)                                       RAND0007
    COMMON/NFEED/NSEED                                              RAND0008
    MD=2147483647                                                   RAND0009
    ML=764261123                                                    RAND0010
    FMD=DFLOAT(MD)                                                  RAND0011
    NSEED=NSEED*ML                                                  RAND0012
    IF (NSEED.LT.0) NSEED=NSEED+MD                                  RAND0013
    U=DFLOAT(NSEED)                                                 RAND0014
    U=U/FMD                                                         RAND0015
    RETURN                                                          RAND0016
    END                                                             RAND0017
    SUBROUTINE RHANDS(YMXB,BMAB,BETAO,OBS)                          RHAN0001
C                                                                   RHAN0002
C                                                                   RHAN0003
C     THIS SUBROUTINE FORMS THE RIGHT HAND SIDES FOR THE LINEAR     RHAN0004
C     PROGRAMMING PROBLEM.                                          RHAN0005
C                                                                   RHAN0006
C                                                                   RHAN0007
    IMPLICIT REAL*8 (A-H,O-Z)                                       RHAN0008
    COMMON/VALUE/A(20,10),X(20,10),B(20),MP2,N,NC,NOBS,IP           RHAN0009
    COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6         RHAN0010
    DIMENSION OBS(20),YMXB(20),BMAB(20),BETAO(10)                   RHAN0011
C                                                                   RHAN0012
C    ARRAYS HAVE THE FOLLOWING DIMENSIONS: OBS(NOBS),YMXB(NOBS),    RHAN0013
C    BMAB(NC)                                                       RHAN0014
C                                                                   RHAN0015
C                                                                   RHAN0016
C     CALCULATE THE RHS QUANTITY,(Y-X*BETAO)                        RHAN0017
C                                                                   RHAN0018
    DO 10 I=1,NOBS                                                  RHAN0019
    SUM=OBS(I)                                                      RHAN0020
    DO 20 J=1,IP                                                    RHAN0021
 20 SUM=SUM-X(I,J)*BETAO(J)                                         RHAN0022
 10 YMXB(I)=SUM                                                     RHAN0023
C                                                                   RHAN0024
C     CALCULATE THE RHS QUANTITY, (B-A*BETAO)                       RHAN0025
C                                                                   RHAN0026
    DO 30 I=1,NC                                                    RHAN0027
    SUM=B(I)                                                        RHAN0028
    DO 40 J=1,IP                                                    RHAN0029
 40 SUM=SUM-A(I,J)*BETAO(J)                                         RHAN0030
 30 BMAB(I)=SUM                                                     RHAN0031
    IF(IWRIT1.EQ.0) GO TO 100                                       RHAN0032
    WRITE(6,650) (YMXB(I),I=1,NOBS)                                 RHAN0033
650 FORMAT(1H0,'          THE RIGHT HAND SIDES, Y-X*BETAO',/,       RHAN0034
   *1H ,10X,10F11.5,/,1H ,10X,10F11.5)                              RHAN0035
    WRITE(6,60) (BMAB(I),I=1,NC)                                    RHAN0036
 60 FORMAT(1H0,'          THE RIGHT HAND SIDES, B-A*BETAO',/,       RHAN0037
   *1H ,10X,10F11.5,/,1H ,10X,10F11.5)                              RHAN0038
100 CONTINUE                                                        RHAN0039
    RETURN                                                          RHAN0040
    END                                                             RHAN0041
    SUBROUTINE BTILDE(BETAO,OBS)                                    BTIL0001
C                                                                   BTIL0002
C                                                                   BTIL0003
C     SUBROUTINE BTILDE CALUCULATES THE LEAST SQUARES               BTIL0004
C     ESTIMATE OF BETA, BETAO.                                      BTIL0005
C                                                                   BTIL0006
C                                                                   BTIL0007
    IMPLICIT REAL*8 (A-H,O-Z)                                       BTIL0008
    COMMON/IWRITE/IWRIT1,IWRIT2,IWRIT3,IWRIT4,IWRIT5,IWRIT6         BTIL0009
    COMMON/VALUE/A(20,10),X(20,10),B(20),MP2,N,NC,NOBS,IP           BTIL0010
    DIMENSION BETAO(10),OBS(20),XTX(10,10),XTY(20)                  BTIL0011
C                                                                   BTIL0012
C    ARRAYS HAVE THE FOLLOWING DIMENSIONS: BETAO(IP),OBS(NOBS),      BTIL0013
C    XTX(IP,IP),XTY(NOBS)                                           BTIL0014
C                                                                   BTIL0015
C                                                                   BTIL0016
C    CALCULATE X'X INVERSE.                                         BTIL0017
C                                                                   BTIL0018
    DO 10 I=1,IP                                                    BTIL0019
    DO 20 K=1,IP                                                    BTIL0020
    SUM=0.D0                                                        BTIL0021
    DO 30 J=1,NOBS                                                  BTIL0022
    SUM=SUM+X(J,I)*X(J,K)                                           BTIL0023
 30 CONTINUE                                                        BTIL0024
    XTX(I,K)=SUM                                                    BTIL0025
 20 CONTINUE                                                        BTIL0026
 10 CONTINUE                                                        BTIL0027
    CALL INVERT(XTX,IP)                                             BTIL0028
    IF(IWRIT1.EQ.0) GO TO 100                                       BTIL0029
    WRITE(6,40)                                                     BTIL0030
 40 FORMAT(1H0,'          THE INVERSE OF XTX')                      BTIL0031
    DO 60 I=1,IP                                                    BTIL0032
 60 WRITE(6,50) (XTX(I,K),K=1,IP)                                   BTIL0033
 50 FORMAT(1H ,9X,8F14.6)                                           BTIL0034
100 CONTINUE                                                        BTIL0035
C                                                                   BTIL0036
C     CALCULATE THE LEAST SQUARES ESTIMATE OF BETA, BETAO.          BTIL0037
C                                                                   BTIL0038
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      DO 15 I=1,IP                                                      BTIL0039
      SUM=0.D0                                                          BTIL0040
      DO 25 J=1,NOBS                                                    BTIL0041
   25 SUM=SUM+X(J,I)*OBS(J)                                             BTIL0042
      XTY(I)=SUM                                                        BTIL0043
   15 CONTINUE                                                          BTIL0044
      DO 35 J=1,IP                                                      BTIL0045
      SUM=0.D0                                                          BTIL0046
      DO 45 I=1,IP                                                      BTIL0047
      SUM=SUM+XTX(J,I)*XTY(I)                                           BTIL0048
      BETA0(J)=SUM                                                      BTIL0049
   45 CONTINUE                                                          BTIL0050
   35 CONTINUE                                                          BTIL0051
      IF(IWRIT1.EQ.0) GO TO 105                                         BTIL0052
      WRITE(6,55)                                                       BTIL0053
   55 FORMAT(1H0,'          THE VALUE OF BETA0 TO COMPUTE RHS')         BTIL0054
      WRITE(6,65) (BETA0(J),J=1,IP)                                     BTIL0055
   65 FORMAT(1H ,9X,8F14.6)                                             BTIL0056
  105 CONTINUE                                                          BTIL0057
      RETURN                                                            BTIL0058
      END                                                               BTIL0059
      SUBROUTINE INVERT(A,N)                                            INVR0001
C                                                                       INVR0002
C          THIS SUBROUTINE INVERTS AN N BY N MATRIX                     INVR0003
C                                                                       INVR0004
      IMPLICIT REAL*8 (A-H,O-Z)                                         INVR0005
      DIMENSION A(10,10)                                                INVR0006
   39 DO 51 I=1,N                                                       INVR0007
      IF(A(I,I).EQ.0.D0) A(I,I)=0.1D-30                                 INVR0008
      A(I,I)=1.0D0/A(I,I)                                               INVR0009
      DO 52 J=1,N                                                       INVR0010
      IF(J-I) 53,52,53                                                  INVR0011
53    A(I,J)=A(I,J)*A(I,I)                                              INVR0012
52    CONTINUE                                                          INVR0013
      DO 51 J=1,N                                                       INVR0014
      IF(J-I) 54,51,54                                                  INVR0015
54    DO 56 K=1,N                                                       INVR0016
      IF(K-I) 55,56,55                                                  INVR0017
55    A(J,K)=A(J,K)-A(J,I)*A(I,K)                                       INVR0018
56    CONTINUE                                                          INVR0019
      A(J,I)=-A(J,I)*A(I,I)                                             INVR0020
51    CONTINUE                                                          INVR0021
      RETURN                                                            INVR0022
      END                                                               INVR0023
      SUBROUTINE NORMAL(NOBS,OBS)                                       NORM0001
C                                                                       NORM0002
C                                                                       NORM0003
C          SUBROUTINE NORMAL. . .                                       NORM0004
C           GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS         NORM0005
C           NORMAL (0,1)) USING A RANDOM NORMAL GENERATOR               NORM0006
C           CALLED BUTLER'S ALGORITHM AS ADAPTED FROM RAND.             NORM0007
C                                                                       NORM0008
C                                                                       NORM0009
      DIMENSION RANDM(20)                                               NORM0010
      DIMENSION C(6),X(257),U(3),STORE(256),R(256)                      NORM0011
      COMMON/NFEED/NSEED                                                NORM0012
      DATA C/2.515517,.802853,.010328,1.43279,.189269,.0013081/         NORM0013
      DATA IX,JX,STORE /Z7FFFDEC3,Z7DBD1557,256*0.0/                    NORM0014
      DOUBLE PRECISION OBS(20),UNIF                                     NORM0015
      CONST = SQRT(1./(2.*3.14159))                                     NORM0016
      X(1) =-3.6                                                        NORM0017
      X(257)=3.6                                                        NORM0018
      FOLD=0.0                                                          NORM0019
      RAT=1./256.                                                       NORM0020
      RAN=0.0                                                           NORM0021
      DO 10 I=1,255                                                     NORM0022
      RAN=RAN+RAT                                                       NORM0023
      IF (I.GT.128) GO TO 12                                            NORM0024
      T = SQRT(-2.*ALOG(RAN))                                           NORM0025
      GO TO 14                                                          NORM0026
   12 T = SQRT(-2.*ALOG(1.-RAN))                                        NORM0027
   14 Z = T - (C(1)+C(2)*T+C(3)*T**2)/(1.+C(4)*T+C(5)*T**2+C(6)*T**3)   NORM0028
      IF (I.LT.129) Z=-Z                                                NORM0029
      X(I+1)=Z                                                          NORM0030
      FNEW=CONST*EXP(-Z**2/2.)                                          NORM0031
   20 R(I) = (FNEW-FOLD)/(FNEW+FOLD)                                    NORM0032
   10 FOLD=FNEW                                                         NORM0033
      FNEW=0.0                                                          NORM0034
      R(256) = (FNEW-FOLD)/(FNEW+FOLD)                                  NORM0035
      SUM=0.0                                                           NORM0036
      SS = 0.0                                                          NORM0037
      DO 30 L=1,NOBS                                                    NORM0038
      IX=IX*65539                                                       NORM0039
      JX=JX*262147                                                      NORM0040
      RANDM(L)=.4656613E-9*FLOAT(IABS(IX+JX))                           NORM0041
      I=256.*RANDM(L)+1.0                                               NORM0042
      DO 32 K=1,3                                                       NORM0043
      CALL RAND(UNIF)                                                   NORM0044
   32 U(K)=UNIF                                                         NORM0045
      Z=X(I+1)-X(I)                                                     NORM0046
      IF (U(3).GT.ABS(R(I))) GO TO 34                                   NORM0047
      RANDM(L)=X(I)+Z*U(1)                                              NORM0048
      GO TO 36                                                          NORM0049
   34 IF (R(I).LT.0.0) GO TO 50                                         NORM0050
      RANDM(L)=AMAX1(U(1),U(2))                                         NORM0051
      GO TO 52                                                          NORM0052
   50 RANDM(L)=AMIN1(U(1),U(2))                                         NORM0053
   52 RANDM(L)=X(I)+Z*RANDM(L)                                          NORM0054
   36 CONTINUE                                                          NORM0055
      OBS(L)=RANDM(L)                                                   NORM0056
   30 CONTINUE                                                          NORM0057
      RETURN                                                            NORM0058
      END                                                               NORM0059
      SUBROUTINE DOUBLE(NOBS,DBLEXP)                                    DOUB0001
C                                                                       DOUB0002
C          SUBROUTINE DOUBLE . . .                                      DOUB0003
C            GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS        DOUB0004
C            DOUBLE EXPONENTIAL VARIABLES WITH MEAN ZERO AND            DOUB0005
C            VARIANCE ONE.)                                             DOUB0006
C                                                                       DOUB0007
      IMPLICIT REAL*8 (A-H,O-Z)                                         DOUB0008
      COMMON/NFEED/NSEED                                                DOUB0009
      DIMENSION DBLEXP(20),UXX(2)                                       DOUB0010
      DO 10 I=1,NOBS                                                    DOUB0011
      DO 20 J=1,2                                                       DOUB0012
      CALL RAND(UX)                                                     DOUB0013
   20 UXX(J)=UX                                                         DOUB0014
      UW=(1.0D0/DSQRT(2.D0))*(DLOG(UXX(2))-DLOG(UXX(1)))                DOUB0015
   10 DBLEXP(I)=UW                                                      DOUB0016
      RETURN                                                            DOUB0017
```

```
      END                                                           DOUB0018
      SUBROUTINE UNIFRM(NOBS,UVAR1)                                 UNIF0001
C                                                                   UNIF0002
C          SUBROUTINE UNIFRM . . .                                  UNIF0003
C            GENERATES THE EPSILON STAR'S (ERRORS DISTRIBUTED AS    UNIF0004
C            UNIFORM VARIABLES WITH MEAN ZERO, VARIANCE ONE.)       UNIF0005
C                                                                   UNIF0006
      IMPLICIT REAL*8 (A-H,O-Z)                                     UNIF0007
      COMMON/NFEED/NSEED                                            UNIF0008
      DIMENSION UVAR1(20)                                           UNIF0009
      DO 10 I=1,NOBS                                                UNIF0010
      CALL RAND(UZO)                                                UNIF0011
   10 UVAR1(I)=UZO*DSQRT(12.D0)                                     UNIF0012
      RETURN                                                        UNIF0013
      END                                                           UNIF0014
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The parameters in a linear regression model can be estimated by minimizing the sum of the absolute residuals (L_1 estimation) instead of the more classical approach of minimizing the sum of squared residuals (least squares estimation). In addition to other nice properties L_1 estimators are less sensitive to outliers than least squares estimators. This paper describes a linear programming algorithm and computer program for obtaining L_1 estimators and estimates of their covariances when the regression parameters are restricted to satisfy specified linear constraints. These estimated

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19. (cont.)

variances and covariances of L_1 estimators
computer algorithm
least squares
Monte Carlo

20. (cont.)

covariances are the new feature in this work and are an extremely important ingredient in hypothesis tests and confidence interval construction. Technical Report 64 describes a similar procedure for obtaining unbiased L_1 estimators when there are no constraints on the parameters.

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